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On the quality of orthometric correction determination

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Abstract: In the present study, a comparison of accuracy and precision between two formulae for assessing Helmert's orthometric correction was carried out. Two test levelling lines were used for comparison. For the two lines, the actual levelling and gravity data were used as reference (real world) data, whereas respective UCPH2002 derived gravity values and SRTM30-interpolated elevation data were executed as erroneous input information. Practically, one formula proved to be less sensitive to the propagation of biases and random errors of the input data, thus yielding a remarkably accurate and precise orthometric correction. So, it is recommended to use this formula, especially when using interpolated end benchmarks elevations in the computation of the height correction. Furthermore, the determination of realistic uncertainties of the height corrections could help model them as observed parameters during the adjustment of levelling loops.

Keywords: Orthometric correction, Error analysis, Uncertainty, Accuracy

1. Introduction

Orthometric height (*OH*) is of a great importance for engineering and geophysical applications. Orthometric height differences are computed through the addition of the orthometric correction (*OC*) to the levelled height differences along the levelling lines. Such correction compensates for the error arising from the non-parallel geo-potential surfaces. So, via the application of *OC*, spirit levelling loop closures would theoretically vanish (Sanso and Vaniček, 2006). For this purpose, gravity information should be available at appropriate sections along the levelling route (Heiskanen and Moritz, 1967).

The impact of the accuracies of both gravity data and benchmarks' heights, on the computed *OC*, was previously studied by Filmer and Featherstone (2011). In this respect, besides Helmert *OC*, other two types of height corrections were considered. Also, the use of global geo-potential models derived gravity in Helmert *OC* assessment was investigated by Filmer et al. (2013) and Hassouna (2013). Regarding Helmert *OC* accuracy, such studies considered the common *OC* formula derived by Heiskanen and Moritz (1967). Also, no explicit investigation has been carried out on the *OC* precision, as expressed by its variance.

Hwang and Hsiao (2003) introduced a new formula for the computation of Helmert *OC*. However, the qualities of *OC*, as computed from the two formulae, have never been compared. Such comparison could flag the specific formula that is more accurate and/or more precise. Obviously, an accurate *OC* model would be more efficient if the input gravity and/or benchmark elevations exhibit some biases. A good example for gravity bias is the omission error of a geo-potential model, from which gravity data are derived. On the other hand, a more precise *OC* implies an optimal propagation of the input data random error through the *OC* model. While an accurately computed *OC* value is directly related to its nature as a correction for a systematic error, it could be claimed that the precision (or uncertainty) of *OC* is less stringent. However, such uncertainty expresses the spread of the computed *OC* about its computed value.

Motivated from the above, the objective of the current study is to compare the above two Helmert *OC* formulae, regarding the accuracy and precision of the resulting *OC*. Such investigation will comprise two levelling lines in Egypt with observed elevations and gravity data. The first line runs along the Nile valley, while the second is located in the Western desert. The two lines exhibit relatively moderate and mountainous terrain roughness, respectively. The qualities of geo-potential model derived gravity and digital terrain model (DTM) derived benchmark elevations are used for a practical comparison among the two formulae over the levelling lines. In this respect, the UCPH2002 geo-potential model (Howe and Tscherning, 2002) is used, up to degree and order 90. Also, the SRTM30 global elevation model (USGS, 2006) is utilized.

2. OC error analysis: comparative algorithms

The *OC* along a spirit levelling line, AB, is commonly expressed by (Heiskanen and Moritz, 1967)

$$OC = \sum_{i=1}^{k} \left(\frac{g_i}{\gamma_0} - 1 \right) \Delta n_i + \left(\frac{\bar{g}_A}{\gamma_0} - 1 \right) H_A - \left(\frac{\bar{g}_B}{\gamma_0} - 1 \right) H_A , \quad (1)$$

where

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Δn_i the geometric height increment relevant to the ith levelling section,

k the number of levelling sections,

g_i the observed gravity relevant to the ith section,

γ_0 the normal gravity at geodetic latitude 45° on the WGS-84 ellipsoid, which can be computed according to Moritz (1980),

H_A the elevation of the start benchmark A,

H_B the elevation of the end benchmark B,

$\bar{g}_A$ the mean gravity along the plumb line at A $\left(\bar{g}_A = g_A + 0.0424 H_A\right)$,

$\bar{g}_B$ the mean gravity relevant to B $\left(\bar{g}_B = g_B + 0.0424 H_B\right)$,

g_A & g_B the observed gravity at A and B, respectively.

Equivalently, but alternatively expressed, the *OC* could be formulated as follows (Hwang and Hsiao, 2003)

$$OC = \sum_{i=1}^{k} \left(\frac{g_i}{\bar{g}_B} - 1 \right) \Delta n_i + \left(\frac{\bar{g}_A}{\bar{g}_B} - 1 \right) H_A . \qquad (2)$$

Based on the different functional models in Eqs. (1) and (2), one could in general expect different error propagation characteristics. The following two sub-sections will explore both the systematic and random error analysis features for OC as estimated from Eqs. (1) and (2).

2.1 OC systematic error analysis

In the sense of Schofield and Breach (2007, Eq. 2.6 therein), it might be recognized that the resultant systematic error in *OC* could be assessed simply via the total differential. Thus, applying the total differential operator to Eq. (1), it follows that the resultant systematic error in *OC*, δOC , may be expressed as

$$\delta OC = \sum_{i=1}^{k} \frac{\partial OC}{\partial g_i} \delta g_i + \sum_{i=1}^{k} \frac{\partial OC}{\partial \Delta n_i} \delta \Delta n_i + \frac{\partial OC}{\partial \bar{g}_A} \delta \bar{g}_A + \frac{\partial OC}{\partial \bar{g}_B} \delta \bar{g}_B + \frac{\partial OC}{\partial H_A} \delta H_A + \frac{\partial OC}{\partial H_B} \delta H_B , \qquad (3)$$

where δg_i , $\delta \Delta n_i$, $\delta \bar{g}_A$, $\delta \bar{g}_B$, δH_A and δH_B denote the systematic errors in the respective input quantities. The partial derivatives and $\delta \bar{g}_A$, $\delta \bar{g}_B$ in Eq. (3) can be derived as follows (Filmer and Featherstone, 2011)

$$\frac{\partial OC}{\partial g_i} = \frac{\Delta n_i}{\gamma_0}, \qquad (4a)$$

$$\frac{\partial OC}{\partial \Delta n_i} = \frac{g_i}{\gamma_0} - 1, \qquad (4b)$$

$$\frac{\partial OC}{\partial \bar{g}_A} = \frac{H_A}{\gamma_0}, \qquad (4c)$$

$$\frac{\partial OC}{\partial \bar{g}_B} = -\frac{H_B}{\gamma_0}, \qquad (4d)$$

$$\frac{\partial OC}{\partial H_A} = \frac{\bar{g}_A}{\gamma_0} - 1, \qquad (4e)$$

$$\frac{\partial OC}{\partial H_B} = -\left(\frac{\bar{g}_B}{\gamma_0} - 1 \right), \qquad (4f)$$

$$\delta \bar{g}_A = \delta g_A + 0.0424 \delta H_A , \qquad (4g)$$

$$\delta \bar{g}_B = \delta g_B + 0.0424 \delta H_B . \qquad (4h)$$

So, Eq. (3) may be re-written as follows

$$\delta OC = \sum_{i=1}^{k} \frac{\Delta n_i}{\gamma_0} \delta g_i + \sum_{i=1}^{k} \left(\frac{g_i}{\gamma_0} - 1 \right) \delta \Delta n_i + \frac{H_A}{\gamma_0} \delta \bar{g}_A - \frac{H_B}{\gamma_0} \delta \bar{g}_B + \left(\frac{\bar{g}_A}{\gamma_0} - 1 \right) \delta H_A - \left(\frac{\bar{g}_B}{\gamma_0} - 1 \right) \delta H_B . \qquad (5)$$

On the other hand, the total differential of Eq. (2) can be expressed by

$$\delta OC = \sum_{i=1}^{k} \frac{\partial OC}{\partial g_i} \delta g_i + \sum_{i=1}^{k} \frac{\partial OC}{\partial \Delta n_i} \delta \Delta n_i + \frac{\partial OC}{\partial \bar{g}_A} \delta \bar{g}_A + \frac{\partial OC}{\partial \bar{g}_B} \delta \bar{g}_B + \frac{\partial OC}{\partial H_A} \delta H_A , \qquad (6)$$

with

$$\frac{\partial OC}{\partial g_i} = \frac{\Delta n_i}{\bar{g}_B}, \qquad (7a)$$

$$\frac{\partial OC}{\partial \Delta n_i} = \left(\frac{g_i}{\bar{g}_B} - 1 \right), \qquad (7b)$$

$$\frac{\partial OC}{\partial \bar{g}_A} = \frac{H_A}{\bar{g}_B}, \qquad (7c)$$

$$\frac{\partial OC}{\partial \bar{g}_B} = -\frac{1}{\bar{g}_B^{\,2}} \left[\sum_{i=1}^{k} g_i \Delta n_i + H_A \bar{g}_A \right], \qquad (7d)$$

$$\frac{\partial OC}{\partial H_A} = \left(\frac{\overline{g}_A}{\overline{g}_B} - 1 \right). \tag{7e}$$

Accordingly, Eq. (6) may be formulated as follows

$$\delta OC = \sum_{i=1}^{k} \frac{\Delta n_i}{\overline{g}_B} \delta g_i + \sum_{i=1}^{k} \left(\frac{g_i}{\overline{g}_B} - 1 \right) \delta \Delta n_i + \frac{H_A}{\overline{g}_B} \delta \overline{g}_A - \frac{1}{\overline{g}_B^{\,2}} \left[\sum_{i=1}^{k} g_i \Delta n_i + H_A \overline{g}_A \right] \delta \overline{g}_B + \left(\frac{\overline{g}_A}{\overline{g}_B} - 1 \right) \delta H_A . \tag{8}$$

2.2 OC random error analysis

The impact of random errors on *OC* follows from the application of the law of variance-covariance propagation to Eqs. (1) and (2). So, neglecting the error covariances and applying the error propagation principle to Eq. (1), the *OC* variance, $\sigma^2{}_{OC}$, is expressed by (e.g. Ghilani and Wolf, 2006)

$$\sigma^2{}_{OC} = \sum_{i=1}^{k} \left(\frac{\partial OC}{\partial g_i} \right)^2 \sigma^2{}_{g_i} + \sum_{i=1}^{k} \left(\frac{\partial OC}{\partial \Delta n_i} \right)^2 \sigma^2{}_{\Delta n_i} + \left(\frac{\partial OC}{\partial \overline{g}_A} \right)^2 \sigma^2{}_{\overline{g}_A} + \left(\frac{\partial OC}{\partial \overline{g}_B} \right)^2 \sigma^2{}_{\overline{g}_B} + \left(\frac{\partial OC}{\partial H_A} \right)^2 \sigma^2{}_{H_A} + \left(\frac{\partial OC}{\partial H_B} \right)^2 \sigma^2{}_{H_B}, \tag{9}$$

where σ_{g_i}, $\sigma_{\Delta n_i}$, $\sigma_{\overline{g}_A}$, $\sigma_{\overline{g}_B}$, σ_{H_A} and σ_{H_B} stand for the error standard deviations relevant to the input quantities, which reflects their precisions. The partial derivatives in Eq. (9) are the same derived in Eq. (4). So, Eq. (9) can be written as follows

$$\sigma^2{}_{OC} = \sum_{i=1}^{k} \left(\frac{\Delta n_i}{\gamma_0} \right)^2 \sigma^2{}_{g_i} + \sum_{i=1}^{k} \left(\frac{g_i}{\gamma_0} - 1 \right)^2 \sigma^2{}_{\Delta n_i} + \left(\frac{H_A}{\gamma_0} \right)^2 \sigma^2{}_{\overline{g}_A} + \left(\frac{H_B}{\gamma_0} \right)^2 \sigma^2{}_{\overline{g}_B} + \left(\frac{\overline{g}_A}{\gamma_0} - 1 \right)^2 \sigma^2{}_{H_A} + \left(\frac{\overline{g}_B}{\gamma_0} - 1 \right)^2 \sigma^2{}_{H_B}. \tag{10}$$

Similarly, applying the law of variance-covariance propagation to Eq. (2),

$$\sigma^2{}_{OC} = \sum_{i=1}^{k} \left(\frac{\partial OC}{\partial g_i} \right)^2 \sigma^2{}_{g_i} + \sum_{i=1}^{k} \left(\frac{\partial OC}{\partial \Delta n_i} \right)^2 \sigma^2{}_{\Delta n_i} + \left(\frac{\partial OC}{\partial \overline{g}_A} \right)^2 \sigma^2{}_{\overline{g}_A} + \left(\frac{\partial OC}{\partial \overline{g}_B} \right)^2 \sigma^2{}_{\overline{g}_B} + \left(\frac{\partial OC}{\partial H_A} \right)^2 \sigma^2{}_{H_A}, \tag{11}$$

which after using Eq. (7) gives

$$\sigma^2{}_{OC} = \sum_{i=1}^{k} \left(\frac{\Delta n_i}{\overline{g}_B} \right)^2 \sigma^2{}_{g_i} + \sum_{i=1}^{k} \left(\frac{g_i}{\overline{g}_B} - 1 \right)^2 \sigma^2{}_{\Delta n_i} + \left(\frac{H_A}{\overline{g}_B} \right)^2 \sigma^2{}_{\overline{g}_A} + \left[\frac{1}{\overline{g}_B^{\,2}} \left[\sum_{i=1}^{k} g_i \Delta n_i + H_A \overline{g}_A \right] \right]^2 \sigma^2{}_{\overline{g}_B} + \left(\frac{\overline{g}_A}{\overline{g}_B} - 1 \right)^2 \sigma^2{}_{H_A}. \tag{12}$$

Also, using the law of error propagation, it follows that

$$\sigma^2{}_{\overline{g}_A} = \sigma^2{}_{g_A} + (0.0424)^2 \sigma^2{}_{H_A}, \tag{13a}$$

$$\sigma^2{}_{\overline{g}_B} = \sigma^2{}_{g_B} + (0.0424)^2 \sigma^2{}_{H_B}. \tag{13b}$$

3. Input data and input errors for the test levelling lines

The current study considers two levelling lines in Egypt with observed elevations and gravity data. The first line (Line I) runs along the Nile valley, while the second (Line II) is located in the Western desert. Line I and line II exhibit relatively moderate and mountainous terrain roughness, respectively. Figure 1 depicts two post maps for the gravity points along both lines. While the available terrestrial gravity and elevation data are used as real world reference values in the current investigation, the input "erroneous" gravity and height information (input into the error analysis algorithms) are derived from the UCPH2002 geo-potential model and the SRTM30 terrain model, respectively. First, geo-potential model derived surface gravity was derived as the sum of the UCPH2002-synthesized surface gravity disturbances relative to WGS-84 and the respective normal gravity values (Forsberg and Tscherning, 2008; Filmer and Featherstone, 2011; Filmer et al., 2013; Hassouna, 2013). On the other hand, the SRTM30 grid was used to interpolate respective elevation data at the points of lines I and II, via the B-spline interpolation algorithm (Cimmery, 2010).

Table 1 lists the different features of the two levelling lines under consideration. Table 2 shows the statistics of the differences among the derived "erroneous" gravity and benchmark elevation values; and the respective reference values over the investigated lines.

4. Numerical error analyses comparison

Regarding *OC* accuracy, as given by Eqs. (5) and (8), the section-wise varying δg_i in Table 2 were used to represent the respective gravity biases, while δg_A and δg_B were supposed to be the gravity biases at the start and end benchmark, respectively. While such gravity biases represent the geo-potential model omission error, it could be comparable to gravity biases that could arise from the interpolation of the respective gravity values from a specific gravity data base. In the same manner, δH_A and δH_B (in Table 2) were used as the elevation biases relevant to the start and end benchmarks. Such

biases might seem realistic for the actual lines I and II, provided that the typical surveyor, although could perform a precise spirit levelling, might not have access to the official elevations for all benchmarks in a levelling network. Accordingly, a way out is to use interpolated elevations for the assessment of *OC* (Filmer and Featherstone, 2011).

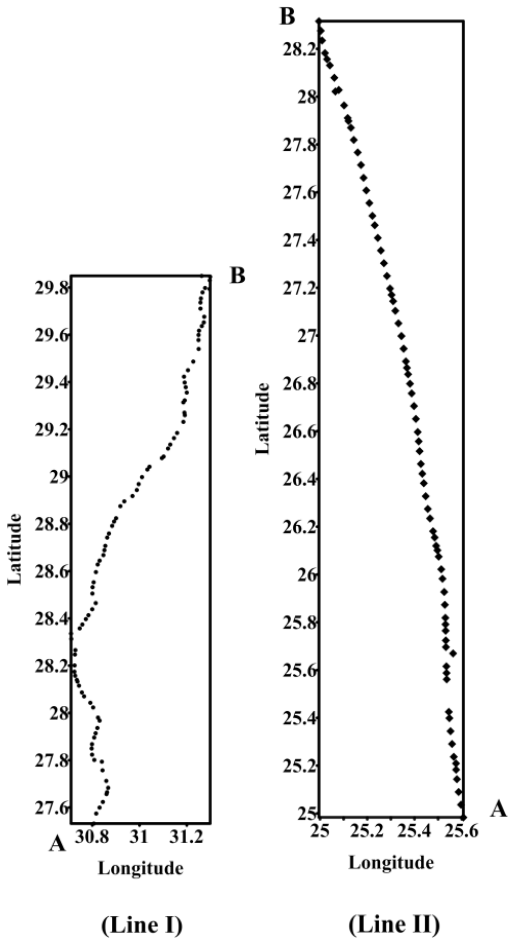

Figure 1: Post maps for Lines (I) and (II)

Table 1: Different features of the two levelling profiles

Line	No. of gravity sections	ΔH (m)	Mean elevation (m)	Length (km)
I	93	-24.68	34.85	280.4
II	78	-391.82	268.18	369.2

The observational noises of the observed (reference) gravity data for the test lines are in the order of 1 mgal. Also, the error budget for the reference elevations of the start and end benchmarks could be as worse as a few centimeters. However, such small uncertainties might not represent those of any eventual interpolated gravity or elevation values, which are greatly affected by the local gravity and terrain signal roughness. So, it was decided to use the spatial standard deviations of the gravity discrepancies, $\sigma_{\delta g_i}$, as representing σ_{g_i} over each levelling line, as given in Table 2.

Similarly, the spatial precision of the elevations discrepancies in Table 2, $\sigma_{\delta H_i}$, was used to represent both σ_{H_A} and σ_{H_B} for each levelling line. Such values were used to evaluate the *OC* precision, σ_{OC}, using Eqs. (10) and (12) for the two lines. Also, as far as the aim of the current work is to compare the accuracy and precision of two formulae in furnishing the *OC*, the very pessimistic biases and standard deviations in Table 2 could help test the sensitivity of the two formulae to systematic and random error propagation. In particular, interpolated benchmark elevations and synthesized or interpolated gravity data could exhibit such bad qualities. It should be emphasized that spirit levelling is characterized by high quality height increments. So, the height increments, Δn_i, were considered errorless over the whole computations.

During the assessment of the *OC* quality features, using Eqs. (5), (8), (10) and (12), the reference gravity, elevations and height increment values were used as the evaluation points of the respective partial derivatives. Also, it should be kept in mind that the input biases and uncertainties of gravity and benchmark elevations were marginally treated. In other words, as will be depicted by Tables 3 and 4, if the gravity accuracy or precision is emphasized, the benchmark elevations are assumed errorless and vice versa. Such separation in error analysis could enable individual investigations of the effect of gravity and benchmark elevations' qualities on that of the *OC*.

It is worth mentioning that $\left(\delta\overline{g}_A \,\&\, \delta\overline{g}_B\right)$ and $\left(\sigma\overline{g}_A \,\&\, \sigma\overline{g}_B\right)$ were appropriately computed from Eqs. (4g & 4h) and (13), respectively, taking into account whether or not the gravity or benchmark elevations are kept errorless.

In the above sense, Tables 3a and 3b summarize the accuracies of *OC*, as computed from Eq. (5) and Eq. (8). Also, Tables 4a and 4b show the respective items, but regarding the precisions of *OC* determination obtained via Eqs. (10) and (12).

5. Discussion and concluding remarks

Tables 3a and 3b show that in general, the accuracies of *OC* determination by the two formulae deteriorate as the terrain roughness increases. Table 3a implies that Eqs. (1) and (2) yield equally accurate *OC* values, if one deals with highly accurate (i.e. nearly errorless) benchmark elevations. Table 3a shows also that in such case, geo-potential models derived gravity could safely be used to assess quite accurate *OC* values in case of moderate terrain roughness, as that represented by line I. This is easy to conclude from the respective two zero

values for δOC . Numerically, this can be attributed to the insignificance of the non-vanishing terms in Eqs. (5) and (8) for line I. On the other hand, Table 3b shows a dramatic deterioration of *OC* accuracy, if computed via Eq. (1), compared to that assessed by Eq. (2). Regarding line I, such deterioration is less pronounced. So, in general, Eq. (2) yield more accurate *OC* magnitudes, if interpolated benchmark heights are to be used along with highly accurate gravity data.

Table 2: Statistics of the discrepancies among the erroneous and reference quantities for the two lines

Line	Item	Unit	$Mean\ \delta g_i$	$\sigma_{\delta g_i}$	$Min.\ \delta g_i$	$Max.\ \delta g_i$	δg_A	δg_B
I	$(g_{UCPH\,2002} - g_{observed})$	$(mgal)$	4.65	8.33	-17.46	22.70	11.50	20.78
II			23.79	13.17	3.00	50.43	10.35	17.95
Line	**Item**	**Unit**	$Mean\ \delta H_i$	$\sigma_{\delta H_i}$	$Min.\ \delta H_i$	$Max.\ \delta H_i$	δH_A	δH_B
I	$(H_{SRTM\,30} - H_{observed})$	(m)	-2.26	1.51	-5.71	1.83	-3.34	-2.37
II			-2.66	32.90	-133.39	200.11	14.94	-8.50

Table 3a: Comparison among the *OC* accuracies (gravity accuracy emphasized)

Line	δg_i $(mgal)$	δg_A $(mgal)$	δg_B $(mgal)$	δOC (mm) **(Eq. 5)**	δOC (mm) **(Eq. 8)**
I	Section-wise varying	11.50	20.78	0.0	0.0
II		10.35	17.95	-4.6	-4.6

Table 3b: Comparison among the *OC* accuracies (elevation accuracy emphasized)

Line	δH_A (m)	δH_B (m)	δOC (mm) **(Eq. 5)**	δOC (mm) **(Eq. 8)**
I	-3.34	-2.37	2.0	0.6
II	14.94	-8.50	-40.2	-4.7

Table 4a: Comparison among the *OC* precisions (gravity precision emphasized)

Line	σ_{g_i} $(mgal)$	σ_{g_A} $(mgal)$	σ_{g_B} $(mgal)$	σ_{OC} (mm) **(Eq. 10)**	σ_{OC} (mm) **(Eq. 12)**
I	8.33			0.5	0.5
II	13.17			8.3	8.3

Table 4b: Comparison among the *OC* precisions (elevation precision emphasized)

Line	σ_{H_A} and σ_{H_B} (m)	σ_{OC} (mm) **(Eq.10)**	σ_{OC} (mm) **(Eq. 12)**
I	1.51	3.1	0.3
II	32.90	78.7	11.3

Regarding *OC* uncertainty, almost all the above comments relevant to Tables 3a and 3b still apply for Tables 4a and 4b. So, Eq. (2) is more precise than Eq. (1), when interpolated benchmark elevations are used. So, generally, for a high quality *OC* determination, it is recommended to use Eq. (2). Such results should be understood to hold to the investigated interpolated benchmark elevations and synthesized gravity values along levelling lines. Besides being more economic, such levelling data sources may be the only available tool for assessing *OC*. The use of the SRTM30 and the UCPH2002 low resolution models in the current study was for the sake of handling an instance worst case. A future work may test other high resolution DTMs and geo-potential models for representing erroneous data.

A further related criterion could be the assessment of the values and qualities of *OCs* for height differences along profiles, which are extracted from DEMs. Such terrain models may be derived from satellite imagery or photogrammetry. In this respect, the published uncertainty of elevations would often be of the order of that discussed in the current study. Accordingly, Eq. (2) is expected to show better error propagation characteristics.

Finally, it could seem rational and innovative to model the *OCs* as biases over levelling circuits in vertical networks. In this sense, *OCs* over levelling lines might be treated as observed weighted parameters during the adjustment of levelling nets (e.g. Ghilani and Wolf, 2006). Of course, the weights of such parameters should lean on the respective error standard deviations, and hence, the importance of *OC* precision may arise. This in turn could optimally help pick out the random loop closures parts during the adjustment.

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