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A linear regression relation between photogrammetric absolute orientation accuracy and ground control distribution

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Abstract: One of the decisive factors of accuracy of absolute orientation in photogrammetry is the distribution of the ground control. Distribution of control is characterized by area within control points and distances between control points in both x and y-directions and hence the strength of geometric figure formed by the control points. This paper reports on research carried out to compare accuracy results of absolute orientation determined using various sets of three ground control points falling at ground positions that form different distributions and hence different enclosed areas. Test results show that best results in both planimetry and height can be obtained when the area formed by the control points is the largest [when enclosed area is 863 ha, the root mean square error (RMSE) in planimetry and height were respectively, ± 0.07m and ± 0.04m. and when enclosed area between used control points was only 20 ha, the RMSE of planimetry and height are ±0.34m and ±0.32m., respectively]. Statistical analysis of results show that variation of diagonal distance between control points is the cause of 80% of the variation of height accuracy and 70% of the variation of the planimetric accuracy. Area enclosed within control points is the cause of 58% of variation in height accuracy and 65% of variation in planimetric accuracy. A linear regression relation can be formed between diagonal distance and planimetric accuracy and height accuracy on one hand and between enclosed area and planimetric accuracy and height accuracy on the other hand. In all cases relationship is formed with 95% confidence level.

Keywords: Photogrammetry, Ground control, Accuracy, Height, Planimetry, Statistical, Regression

1. Introduction

The geospatial information is extensively used in everyday life through aerial photography, satellite imageries and geographical information systems. In addition to general public, professionals concerned with earth and environmental sciences use such information. The most important pillars to these interests are the sciences of spatial information: collection and management. One important source of such information is the overlapping photography that leads to stereoscopic models used in topographic and three dimensional (or 3-D) data production (Elnima, 2015).

Overlap is defined as the amount by which one photograph includes the area covered by the succeeding photograph and is expressed as a percentage. The photo survey plan is usually designed to acquire 50-70 per cent forward overlap between photos along the same flight line and 20-40 per cent lateral overlap between photos on adjacent flight lines (Wolf et al., 2014). In analog instruments stereoscopic model is the three-dimensional view that results when two overlapping photos are viewed and parallax is removed at standard six locations, in an iterative procedure known as relative orientation. Each photograph of the stereo pair provides a slightly different view of the same area. The brain combines both scenes and interprets them as a 3-D view. In analytical and digital instruments, relative orientation process for producing the stereo-model is done through computer processing of readings of coordinates at the six standard model points. The stereoscopic model formed after relative orientation is at an arbitrary model space and is not oriented or scaled to the ground control or map datum. It is transformed to a 3-D ground coordinate system for measurements and map plotting. This process of transformation is known as absolute orientation. It is in this step that the ground control is incorporated into the process. In almost all photogrammetric literature, it is stated that ground control points should be well distributed to cover the model space and hence produce high accuracy, without any quantitative analysis (Mikhail et al., 2001; Linder, 2006; Kraus, 2007; McGlone, 2013; Wolf et al., 2014).

The objective of this work is to investigate the analytical relation between the achieved accuracy (both in height and planimetry) and the distribution of control points. Control distribution is varied to test for different values of area enclosed by control points as well as for different diagonal distances between control points. Linear regression relation between control distribution and absolute orientation accuracy are also formed.

2. Photogrammetric control

One important aspect of the photogrammetric process is control, which is used to establish the position and orientation of the camera at the instant of exposure, the transformation of model space coordinates to ground coordinates, rectification, controlled mosaicking and aerial triangulation. The necessity, accuracy and the rigor of photogrammetric control depend on the particular product sought (Ahmadabadian et al., 2014). Ground control points to be used in photogrammetric operations can be acquired using any of the following three methods: 1-Classical ground survey methods including traversing, triangulation, trilateration, leveling, total station and GPS; 2-Aerial triangulation: bundle or independent models; 3-Kinematic GPS technique where the exterior orientation elements of the camera are determined without ground control. Photogrammetric control points are usually spaced widely around the project area. For large projects, this spacing could be extensive enough to require a significant surveying effort. Therefore, GPS is the better suited surveying method for most large photogrammetric projects. Photo-identifiable control points are established on permanently fixed objects and on flat ground whenever practicable. They should be maintained at the maximum intervals (Wolf et al., 2014; McGlone, 2013).

3. Absolute orientation

With absolute orientation we refer to the process of orienting a stereo model to fit with the ground control system. This is actually a very straightforward process which is usually referred to as the 7-parameter transformation. Note that the 7 parameter transformation establishes the relationship between two 3-D Cartesian coordinate systems (Remondino et al., 2008). These are the model coordinate system in which three dimensional model coordinates of model points formed by relative orientation of overlapping photographs can be extracted and the ground control system in which ground coordinates of control points should be given.

The transformation problem can only be solved if a priori information about some of the parameters is introduced. This is usually done by providing control points. The analytical absolute orientation process is explained in various books of photogrammetry (Schenk, 2001; Schindler, 2014; Xu et al., 2013; Wolf et al., 2014). It is briefly outlined in the following paragraph.

The following vector equation relates the model to the ground coordinate system (Schenk, 2001).

$$\mathbf{P} = \mathbf{s}\,\mathbf{R}\,\mathbf{p_m} - \mathbf{t}$$

where, $\mathbf{P} = [X, Y, Z]^T$ is the vector in the ground coordinate system pointing to the object point P; $\mathbf{p_m} = [x, y, z]^T$is the point vector in the model coordinate system; $\mathbf{t} = [Xt, Yt, Zt]^T$ is the translation vector between the origins of the two coordinate systems; **R** is therotation matrix thatrotates vector $\mathbf{p_m}$into the ground coordinate system; **s** is the scale factor that scales the model to the ground system.

The 7 parameters to be determined comprise 3 rotation angles of the orthogonal rotation matrix **R**, 3 translation parameters **t,** and one scale factor **s:** i.e. (ω, φ, κ, Xt,Yt,Zt, s). In order to solve for these 7 parameters at least seven equations must be available. For example, 2 full control points (yielding 6 equations) and one elevation control point (yielding one equation) would render a solution. If more equations are available then the problem of determining the parameters can be dealt with using a least-squares adjustment. Here, the idea is to minimize the discrepancies between the transformed and the available control points.

The analytical determination of absolute orientation in this research makes use of three ground control points in the photographed object. One surplus equation would allow use of least squares method.

4. Methodology

A pair of overlapping aerial photographs of scale 1:40000 exposed with aerial camera of focal length 152.770mm from an altitude of 6111m above ground surface, covering a rugged ground area in Switzerland, made available with an AC-1 Analytical plotter at the photogrammetric laboratory of the civil engineering department at King Saud University were used in the study.

Seventeen points with known three dimensional ground coordinates of accuracy ±0.01m in both planimetry and height were precisely marked on one of the overlapping pairs. After carrying relative orientation, model coordinates were then made available for absolute orientation. An absolute orientation program has already been developed to transform model coordinates to ground coordinates. The seventeen full ground control points, with ground distribution as shown in Fig. 1, were used to form control and test points for the study. Seven configurations of control point distributions that form different positions in the model and different triangular areas were selected as shown in Figs. 2-8 with the data including the enclosed triangular area and longest diagonal distances between control points are given in Table 1.

Table 1: Geometric data of the seven control distribution configurations

Case No.	Enclosed area (ha)	Diagonal distance (m)
1(544,443,309)	728	6070
2(544,309,327)	737	5860
3(424,433,317)	158	2733
4(545,433,315)	159	4310
5(419,442,226)	20	3822
6(443,226,310)	115	5865
7(443,226,328)	254	3908

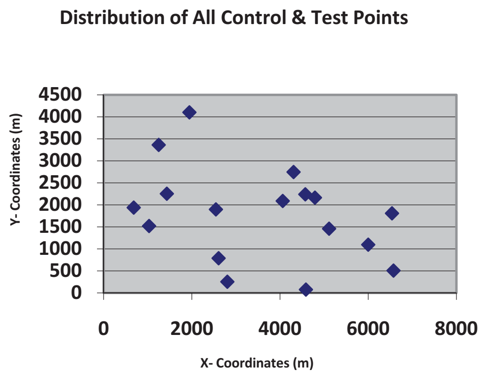

Figure 1: Distribution of all control and test points

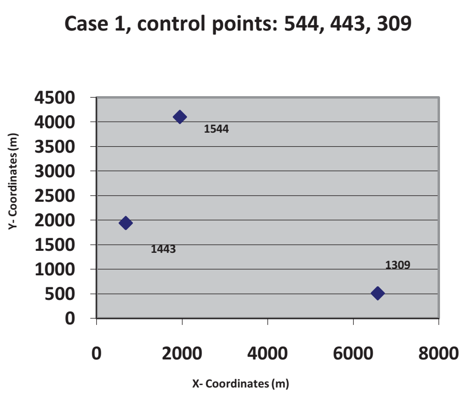

Figure 2: Control points, Case 1

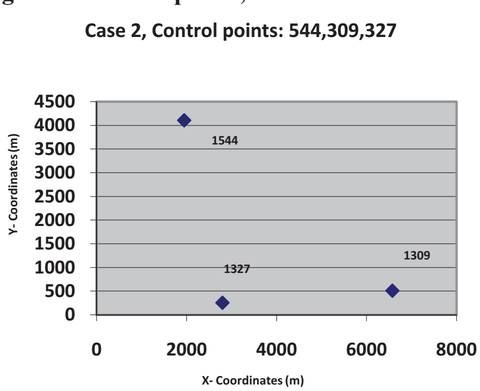

Figure 3: Control points, Case 2

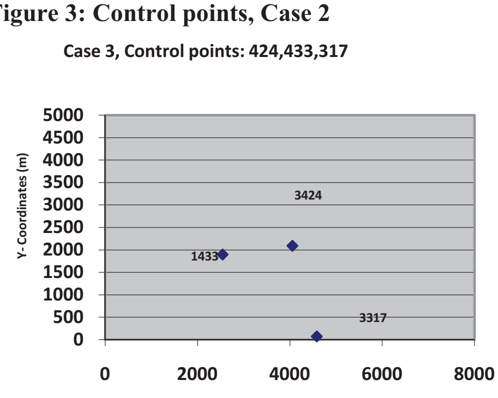

Figure 4: Control points, Case 3

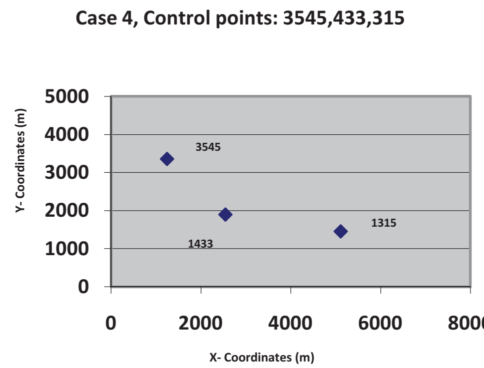

Figure 5: Control points, Case 4

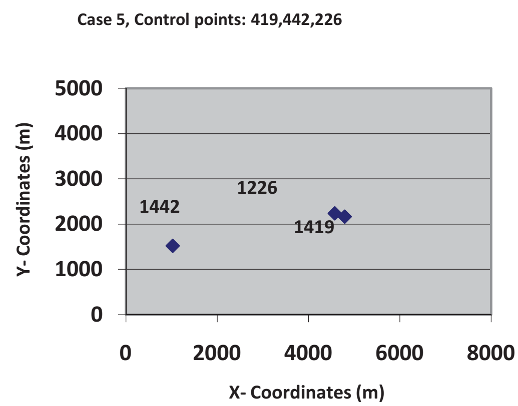

Figure 6: Control points, Case 5

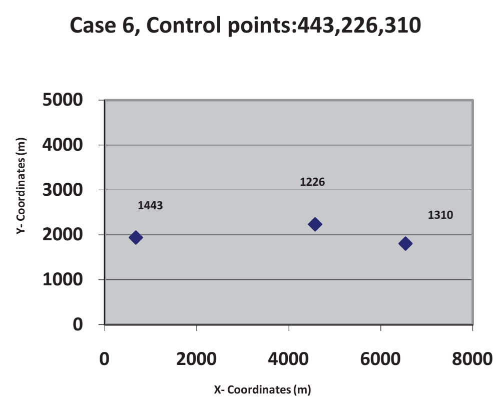

Figure7: Control points, Case 6

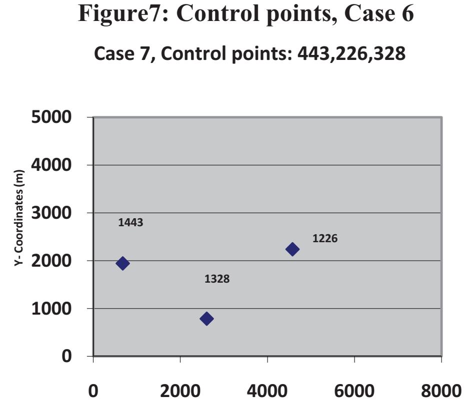

Figure 8: Control points, Case 7

To expresses the relation between obtained accuracy (height or planimetry) and geometric property of control distribution (area enclosed by control points and / or diagonal distance between control points), a simple linear regression model of the form$Y_i = \alpha + \beta X_i + \mu_i$ was modelled(Gujarati and Porter, 2009; Khalil, 2014) where Y_i = the dependent variable (for example, height accuracy, DZ); X_i = the independent variable (area enclosed by control points, A); α = intercept coefficient; β = the slope coeffieint of X_i;μ_i = term of errors.

The following statistical test was performed using EVIEWS statistical software (Griffiths et al., 2008).

H_0: $\beta = 0$ (null hypothesis)

H_1: $\beta \neq 0$ (alternative hypothesis)

5. Results and discussion

The model coordinates of each configuration were processed through the absolute orientation to produce ground coordinates. The obtained ground coordinates from the absolute orientation process were compared to the given coordinates to determine the resulting accuracy in both height and planimetry for each configuration. Results of the seven tested distributions (standard errors in X, Y, P and Z) are given in Table 2 and are shown graphically in Fig. 9 as bar chart and in Fig. 10 as line graph. Fig. 11 shows the results for planimetry, P and height Z in bar chart.

Table 2: Root mean square errors in meters of tested points

Case No.	δX (m)	δY (m)	δP (m)	δZ (m)
1	0.05	0.05	0.07	0.04
2	0.17	0.09	0.19	0.09
3	0.27	0.35	0.44	0.36
4	0.18	0.30	0.40	0.19
5	0.28	0.26	0.34	0.32
6	0.23	0.16	0.26	0.14
7	0.27	0.04	0.27	0.16

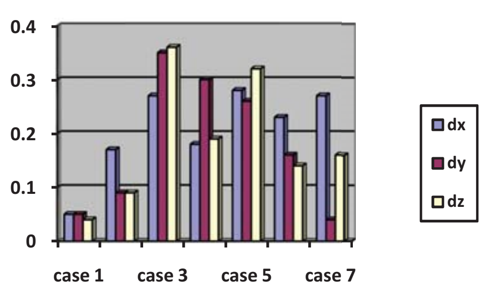

Figure 9: Test results (RMSE in meters in X,Y and Z) shown as bar chart

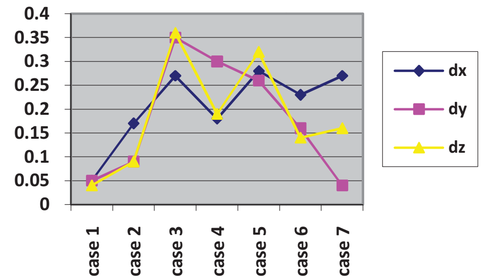

Figure 10: Test results (RMSE in meters) shown as line graph

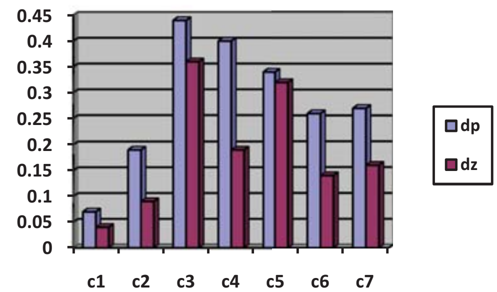

Figure 11: Test results (RMSE in meters) for planimetry and height

The enclosed area, diagonal distance characterizing the geometry of the control points distribution for each case and the resulting planimetric and height accuracy are summarized in Table 3 and Fig. 12.

Table 3: Enclosed area, diagonal distance, planimetric accuracy and height accuracy for the seven cases

Case No.	Enclosed area (ha)	Diagonal distance (m)	δP (m)	δZ (m)
1(544,443,309)	728	6070	0.07	0.04
2(544,309,327)	737	5860	0.19	0.09
3(424,433,317)	158	2733	0.44	0.36
4(545,433,315)	159	4310	0.40	0.19
5(419,442,226)	20	3822	0.34	0.32
6(443,226,310)	115	5865	0.26	0.14
7(443,226,328)	254	3908	0.27	0.16

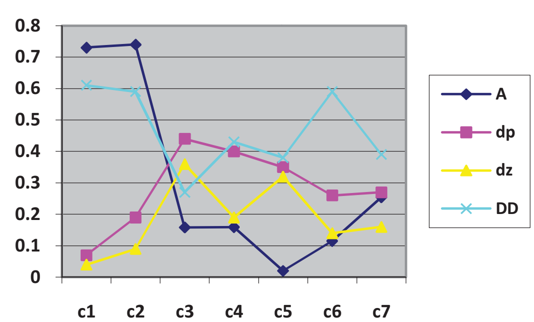

Figure 12: Composite graph showing Area (A, ha x10^3), Diagonal Distance (DD, m), Planimetric error (dp) and height error (dz)

From Table 3 and the graphical result representation of Fig. 12, it can be deduced that the best height and planemetric results are obtained when case 1 is used. This is the case where area enclosed by control points is the largest, distances between control points in both x and y directions are the longest. Next best in accuracy is case 2 where the enclosed area is second largest, distance between control points in x direction is third, (after case 1 and case 7) and distance in y direction is the same as case 1, being better than other cases.

When inspecting Fig. 12, it is clearly noticed that when the area within the control points is large and the diagonal distance is long the planimetric and height accuracy are the best (cases 1 and 2). This agrees well with the common idea of photogrammetrists who recommend that the control points should be well distributed in the model and diagonal distance between control points should be large to allow better absolute orientation. However, there was no scientific rule or statistical study that supported the recommendation. This is why the present statistical study has been carried out.

The data in Table 3 were tested using EVIEWS statistical software (Griffiths et al., 2008).Results are given in Table 4 to Table 7.

Table 4: Results of statistical test for height accuracy and enclosed area (Dependent Variable: DZ; Observations: 7)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.2782	0.0474	5.8660	0.0020
A (Area)	-0.000298	0.0001	-2.6049	0.0480
R^2	0.5758		Mean dependent var	0.1857
Adjusted R^2	0.4909		S.D. dependent var	0.1166
S.E. of regression	0.0832		Sum squared resid	0.0346
F-statistic	6.7858		Prob(F-statistic)	0.0480

The result obtained from the estimated regrssion model shows that the area within control points has a negative relation with the height accuracy (inversely proportional). The coefficient β of the variable Ar, which represents the slope of the regression line is 0.000298. That means if the area within control points is increased by 100 ha the height accuracy will be better by 0.0298m.

The probability of the t-statistic for both coefficeints used in the model is less than 0.05, which means that the null hypothesis is rejected and the alternative hypothesis can be accepted with confidence level of 95%. This indicates that coefficient β is significant and the enclosed area significantly affects the height accuracy. The standard error measures the goodness of fit, its value for coefficient β is 0.000114 showing high precision. The goodness of fit of the model is generally accepted since the value of the R^2 obtained is 0.58. The R^2 value is about 0.58. This also means that 58 percent of the variation in the accuracy of height is due to the area enclosed within control points. The F-statistic has probability smaller than 0.05% which indicates that the general model is significant. Durbin-Watson Statisitics is 2.18 (around 2) showing no correlation between the disturbances.

Table 5: Results of statistical test for planimetric accuracy and enclosed area (Dependent Variable: DP; Observations: 7)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.388050	0.0469	8.2723	0.0004
A (Area)	-0.000344	0.0001	-3.0363	0.0289
R^2	0.6484		Mean dependent var	0.2814
Adjusted R^2	0.5780		S.D. dependent var	0.1267
S.E. of regression	0.0823		Sum squared resid	0.0339
F-statistic	9.2190		Prob(F-statistic)	0.0289

Results indicates that area within control points affects the planimetric accuracy with negative sign (inverse proportion). Standard error of coefficient β of the independent variable (Area) is quite small: 0.0001. Probability value of t-test is also very small being 0.0289 which is less than 5%, and can thus be accepted with confidence level 95%.

The R^2 value is 0.648 which means that almost 65% of variation in planimetric accuracy is due to area enclosed between control points.

Table 6: Results of statistical test for height accuracy and diagonal distance (Dependent Variable: DZ, Observations: 7)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.561312	0.0874	6.4258	0.0014
A (Area)	-8.07E-05	1.82E-05	-4.4391	0.0068
R^2	0.7976		Mean dependen t var	0.1857
Adjusted R^2	0.7571		S.D. dependen t var	0.1166
S.E. of regression	0.0575		Sum squared resid	0.0165
F-statistic	19.7059		Prob(F-statistic)	0.0068

This is a very strong model where the standard error of coefficient β is very small (1.8E-5) and the P-value of the t-test of coefficient β is 0.68% which means confidence level exceeds 99%.

R^2 is 0.7976 indicates that almost 80% of the variation in height accuracy is caused by the change in diagonal distance between control points. P-value of F-statistic is 0.00677, hence the general model: effect of diagonal distance on height accuracy is quite significant.

Table 7: Results of statistical test for planimetrict accuracy and diagonal distance (Dependent Variable: DP, Observations: 7)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.664139	0.1152	5.7627	0.0022
A (Area)	-8.23E-05	2.40E-05	-3.4284	0.0187
R^2	0.7016		Mean dependent var	0.2814
Adjusted R^2	0.6419		S.D. dependent var	0.1267
S.E. of regression	0.0758		Sum squared resid	0.0287
F-statistic	11.7540		Prob(F-statistic)	0.0187

Since P-value of coefficient β is within 2%, model relating diagonal distance between control points and planimetric accuracy is accepted with confidence level 95%.
Since R^2 is 0.7016 then 70% of the variation in planimetric accuracy is due to the diagonal distance between control points.

6. Conclusions

The current study is an attempt to statistically investigate and relate effect of geometric locations of control points (area enclosed between control points and diagonal distance between control points) on height and planimetric accuracy of absolute orientation results. It is found out that both area enclosed by control points and diagonal distance between control points affect the height and planimetric accuracy. It is found that 80% of the variation of height accuracy is due to the variation of the diagonal distance between control points, and 70% of the variation of the planimetric accuracy is due to variation of diagonal distance between control points. Area enclosed within control points is the cause of 58% of variation in height accuracy and 65% of variation in planimetric accuracy.

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