Local commission error analysis of some recent geopotential models: case study in Egypt

Raaed Mohamed Kamel Hassouna
Department of Civil Engineering, Faculty of Engineering in Shebin El-Kom, Minoufiya University - 32511
Shebin El-Kom, Minoufiya, Egypt
Email: rd3165@yahoo.com

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Abstract: In the current work, the local precisions of some recent geopotential models were investigated. This was applied to the Egyptian Territory as an illustrative case study. In particular, computer software was prepared, which computes the global and local uncertainties of the various geopotential model-synthesized anomalous functions, based on the published global noise standard deviations of the relevant coefficients. In this respect, comparisons were held among the global and local uncertainties of the various synthesized gravitational features. Based on the obtained results, it could be concluded that the local uncertainties of the various synthesized gravitational quantities are almost uniform, and typically greater than the relevant global ones. Compared with the noise of some scattered observed data with different types, such uncertainties are significant and should not be neglected. So, it is recommended to take such local model noise into consideration during the evaluation of geopotential harmonic models, with respect to different types of observed gravitational data. Also, the local precision of geopotential models should be launched into the remove-restore technique in local geoid, anomaly or deflection modeling via the various integral methods. Finally, the used mathematical algorithm could be further investigated, regarding its ability to locally examine the signal-to-noise ratio of the harmonic models.

Keywords: Geopotential models, local commission error, local precision, error analysis

1. Introduction

The use of geopotential harmonic models as reference fields in local and regional gravity field modelling has been a standard tool. This could be accomplished via either the least-squares collocation approach or the integral approach. Examples for integral methods are the Stokes, Vening Meinsez, Deflection-Geoid and Hotine's formulas. In modern collocation solution trends, the noise of both the input gravitational observations and the harmonic coefficients of the reference field can be incorporated in the solution (Arabelos, 1989; Tscherning et al., 2001). So, the collocation solutions yield signals' error estimates, which are based on the error budget of both the input gravitational data and the reference field.

Regarding the integral methods, the noise of the input gridded gravitational data could be easily used for the estimation of the precisions of the solutions, via the law of error propagation. However, in these methods, the local precisions (or local commission errors) of the relevant removed geopotential model-derived quantities have not yet be considered. The same comment is valid for any subsequent use of a so-obtained local geoid model, for example, in levelling by GPS.

Therefore, the objective of the current study is to analyze the local commission error behavior of four recent geopotential models. This will be associated with comparisons to the relevant global commission errors. For this purpose, appropriate computer software was prepared. In particular, the software designed for computing the local precision of geopotential models is based on the SPHARM subroutine (Forsberg and Tscherning, 2008). The harmonic models under study are EIGEN-CG01C (Reigber et al., 2006), GGM03C (Tapley et al., 2007), EGM2008 (Pavlis et al., 2008) and ITG-Grace2010 (Mayer-Gürr et al., 2010). Such models have coefficients up to degree and order 360, 360, 2160 and 180, respectively. In the current work, Egypt was selected as the investigation's geographical window. Regarding its global noise behavior, EGM2008 was investigated up to degree 2160. Locally, this model was considered up to degree and order 400 only, as will be discussed in Section 3.

2. Global uncertainties of geopotential models

Table 1 lists the parameters of the harmonic models under consideration. Such models are arranged according to their release date. Among the four models, ITG-Grace2010 is a satellite-only harmonic model.

Table 1: Parameters of the investigated harmonic

Model	Max. Degree	KM ($m^3 s^{-2}$)	Equatorial radius a (m)
EIGEN-CG01C	360	$3.986004415 x 10^{14}$	6378136.46
GGM03C	360	$3.986004415 x 10^{14}$	6378136.30
EGM2008	2160	$3.986004415 x 10^{14}$	6378136.30
ITG-Grace2010	180	$3.986004415 x 10^{14}$	6378136.60

The global commission error of a geopotential model reflects the impact of the noise inherent into the model's coefficients on its behaviour, in a global sense. Cumulative error spectra, which are the sum error degree variances up to a certain degree n, are particularly useful in evaluating the resultant commission error (Tscherning, 1974).

Expressed in terms of the type of the anomalous quantity, as synthesized from a geopotential model, one has geoidal height, gravity anomaly, gravity disturbance and vertical deflections' global commission errors. In spherical approximation, the cumulative error degree variances, EV, of the respective model-derived gravitational features could be expressed as follows

$$EV_N = (R)^2 \sum_{n=0}^{nmax} \sum_{m=0}^{n} (\sigma^2_{Cnm} + \sigma^2_{Snm}), \quad (1)$$

$$EV_{\Delta g} = (KM/R^2)^2 \sum_{n=0}^{nmax} (n-1)^2 \sum_{m=0}^{n} (\sigma^2_{Cnm} + \sigma^2_{Snm}), \quad (2)$$

$$EV_{\delta g} = (KM/R^2)^2 \sum_{n=0}^{nmax} (n+1)^2 \sum_{m=0}^{n} (\sigma^2_{Cnm} + \sigma^2_{Snm}), \quad (3)$$

$$EV_{\xi} = \rho^2 \sum_{n=0}^{nmax} \sum_{m=0}^{n} (n (n+1) - m^2) (\sigma^2_{Cnm} + \sigma^2_{Snm}), \quad (4)$$

$$EV_{\eta} = \rho^2 \sum_{n=0}^{nmax} \sum_{m=0}^{n} m^2 . (\sigma^2_{Cnm} + \sigma^2_{Snm}), \quad (5)$$

with

R the mean radius of the Earth (taken 6371 km),

nmax the maximum degree of the geopotential model,

σ_{Cnm} the error standard deviation of the fully normalized C-coefficient with degree and order n and m,

σ_{Snm} the error standard deviation of the fully normalized S-coefficient with degree and order n and m,

ρ the radian-to-second conversion factor (ρ=206264.8 arc-seconds).

The above relations follow from the law of error propagation as applied to the cumulative degree variances (cumulative power spectra), relevant to the various anomalous quantities (Tscherning, 1974; Jekeli, 1999). For a specific geopotential model, the respective global commission errors (or the global error standard deviations), ESs, could be expressed as follows

$$ES_N = \sqrt{EV_N} \quad (6)$$

$$ES_{E\Delta g} = \sqrt{EV_{\Delta g}} \quad (7)$$

$$ES_{\delta g} = \sqrt{EV_{\delta g}} \quad (8)$$

$$ESE_{\xi} = \sqrt{EV_{\xi}} \quad (9)$$

$$ESE_{\eta} = \sqrt{EV_{\eta}} \quad (10)$$

Table 2 shows the global commission errors of the various gravitational quantities as computed for the four investigated models' uncertainties, based on Eq. (1) to (10). From Table 2, it could be concluded that the ESs relevant to the four models are in general significant, compared to the expected observational noise for the five gravitational quantities.

3. Local noise analysis of harmonic models over Egypt

The various geopotential model-derived gravitational quantities can be computed via the swapped spherical harmonic synthesis as follows (Sneeuw, 1996)

Table 2: Global commission errors of the various gravitational quantities relevant to the four models

Model	N (m)	Δg (mgal)	δg (mgal)	ξ (arc-second)	η (arc-second)
EIGEN-CG01C	0.204	5.397	5.456	0.711	0.722
GGM03C	0.144	5.112	5.154	0.675	0.689
EGM2008 (400)	0.079	1.956	1.978	0.259	0.263
EGM2008 (2160)	0.082	4.229	4.245	0.564	0.603
ITG-Grace2010	0.199	5.270	5.331	0.679	0.658

$$N = (KM/r\gamma) \sum_{m=0}^{Nmax} \sum_{n=m}^{Nmax} (a/r)^n \; [(\bar{C}^*_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \bar{P}_{nm}(\cos\theta)], \quad (11)$$

$$\Delta g = (KM/r^2) \sum_{m=0}^{Nmax} \sum_{n=m}^{Nmax} (n-1) \; (a/r)^n \; [(\bar{C}^*_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \; \bar{P}_{nm}(\cos\theta)], \quad (12)$$

$$\delta g = (KM/r^2) \sum_{m=0}^{Nmax} \sum_{n=m}^{Nmax} (n+1) \; (a/r)^n \; [(\bar{C}^*_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \; \bar{P}_{nm}(\cos\theta)], \quad (13)$$

$$\xi = - (KM/r^2\gamma) \sum_{m=0}^{Nmax} \sum_{n=m}^{Nmax} (a/r)^n \; [(\bar{C}^*_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \; d\bar{P}_{nm}(\cos\theta) /d\theta], \quad (14)$$

$$\eta = - (KM/r^2\gamma.\sin\theta) \sum_{m=0}^{Nmax} m \sum_{n=m}^{Nmax} (a/r)^n \; [(\bar{C}^*_{nm} (-\sin m\lambda) + \bar{S}_{nm} \cos m\lambda) \; \bar{P}_{nm}(\cos\theta)], \quad (15)$$

with

θ the geocentric latitude,

λ the geodetic longitude,

r the geocentric radius,

$\bar{C}^*_{nm}$ the fully normalized spherical harmonic C-coefficients of degree n and order m, reduced for the even zonal harmonics of the reference ellipsoid,

$\bar{S}_{nm}$ the fully normalized spherical harmonic S-coefficients of degree n and order m,

$\bar{P}_{nm}(\sin\theta$ the fully normalized associated Legendre function of degree n and order m.

γ the normal gravity implied by the reference ellipsoid,

$$\gamma = (a\gamma_e \cos^2\varphi + b\gamma p \sin^2\varphi)/\sqrt{(a^2\cos^2\varphi + b^2\sin^2\varphi)}, \quad (16)$$

where a, b are the semi-major and semi-minor axes of the reference ellipsoid; and γ^e and γ^p are the relevant equatorial and polar normal gravity, respectively.

The local uncertainties of the geopotential models describe how the coefficients' noise propagates into the locally synthesized gravimetric features, based on Eqs. (11) to (15). In particular, applying the law of error propagation to Eq. (11) to (15), and neglecting the error covariances among the geopotential coefficients, one obtains the following expressions for the local error variances of the synthesized gravitational quantities

$$\sigma_N^2 = (KM/r\gamma)^2 \sum_{m=0}^{Nmax} \sum_{n=m}^{Nmax} (a/r)^{2n} [(\sigma^2_{Cnm} \cos^2 m\lambda + \sigma^2_{Snm} \sin^2 m\lambda)\ \bar{P}_{nm}^2 (\cos\theta)], \quad (17)$$

$$\sigma_{\Delta g}^2 = (KM/r^2)^2 \sum_{m=0}^{Nmax} \sum_{n=m}^{Nmax} (n-1)^2 (a/r)^{2n} [(\sigma^2_{Cnm} \cos^2 m\lambda + \sigma^2_{Snm} \sin^2 m\lambda)\ \bar{P}_{nm}^2 (\cos\theta)], \quad (18)$$

$$\sigma_{\delta g}^2 = (KM/r^2)^2 \sum_{m=0}^{Nmax} \sum_{n=m}^{Nmax} (n+1)^2 (a/r)^{2n} [(\sigma^2_{Cnm} \cos^2 m\lambda + \sigma^2_{Snm} \sin^2 m\lambda)\ \bar{P}_{nm}^2 (\cos\theta)], \quad (19)$$

$$\sigma_\xi^2 = (KM/r^2\gamma)^2 \sum_{m=0}^{Nmax} \sum_{n=m}^{Nmax} (a/r)^{2n} [(\sigma^2_{Cnm} \cos^2 m\lambda + \sigma^2_{Snm} \sin^2 m\lambda)\ (d\bar{P}_{nm}(\cos\theta)/d\theta)^2], \quad (20)$$

$$\sigma_\eta^2 = (KM/r^2\gamma.\sin\theta)^2 \sum_{m=0}^{Nmax} m^2 \sum_{n=m}^{Nmax} (a/r)^{2n} [(\sigma^2_{Cnm} \sin^2 m\lambda + \sigma^2_{Snm} \cos^2 m\lambda)\ \bar{P}_{nm}^2 (\cos\theta)], \quad (21)$$

where σ^2Cnm and σ^2Snm are defined in Eqs. (1) to (5). The above summation permutation is necessary for an efficient numerical computation of the squares of the solid spherical harmonics in Eq. (17) to (19). Also, this represents a capable tool for the assessment of the squares of the first derivatives of the solid spherical harmonics, with respect to latitude and longitude, as implied by Eqs. (20) and (21), respectively (Tscherning, 1976; Tscherning and Poder, 1982; Holmes and Featherstone, 2002).

Eqs. (17) to (21) were used to evaluate the respective local error variances for the four investigated models at the nodes of a 0.5°x0.5° grid covering the study window (22° N ≤ φ ≤ 32° N; 25° E ≤ λ ≤ 36° E). In regard to the five investigated gravitational quantities, and based on the geographical window under consideration, the above described mathematical algorithm (Eqs. 17 to 21) showed a numerically stable and convergent behavior, up to degree and order of about 400. The numerical tests revealed that this inconvenience was not due to any numerical difficulties during the assessment of the squares of the solid spherical harmonics or the squares of their first derivatives (Holmes and Featherstone, 2002). Particularly, such numerical problem was found to be dependent only on the investigated maximal degree, and so, on the magnitudes of the relevant coefficients' noise. The estimated standard errors of such ultra high-degree harmonic coefficients are rarely accurate due to the limitations in computer power (Jekeli, 1999). So, such result could signal that locally, beyond a resolution of degree and order of about 400, the signal-to-noise ratio of the EGM2008' coefficients begins to be unacceptably small.

So, the investigation of EGM2008 was restricted to that limit, which happened to be comparable with the resolutions of the other three models. This was of benefit for confining the comparisons among the four models to the impact of the coefficients' cumulative error budget rather than the number of coefficients.

Table 3: Statistics of the 0.5°x0.5° local σ_N values relevant to the four models (units: m)

Model	Mean	Standard deviation	Minimum	Maximum
EIGEN-CG01C	0.232	0.003	0.228	0.237
GGM03C	0.174	0.004	0.167	0.181
EGM2008 (400)	0.090	0.001	0.089	0.092
ITG-Grace2010	0.208	0.002	0.206	0.212

Table 4: Statistics of the 0.5°x0.5° local $\sigma_{\Delta g}$ values relevant to the four models (units: mgal)

Model	Mean	Standard deviation	Minimum	Maximum
EIGEN-CG01C	6.343	0.137	6.135	6.584
GGM03C	6.294	0.197	5.992	6.632
EGM2008 (400)	2.320	0.054	2.237	2.415
ITG-Grace2010	5.500	0.064	5.427	5.630

The computed grids of local precisions (or local commission errors) are expressed in terms of the local error standard deviations, σ_N, $\sigma_{\Delta g}$, $\sigma_{\delta g}$, σ_ξ and ση of the anomalous quantities. Tables 3 to 7 show the statistics of such local uncertainties' grids.

Table 5: Statistics of the 0.5°x0.5° local $\sigma_{\delta g}$ values relevant to the four models (units: mgal)

Model	Mean	Standard deviation	Minimum	Maximum
EIGEN-CG01C	6.409	0.137	6.200	6.652
GGM03C	6.345	0.198	6.041	6.686
EGM2008 (400)	2.346	0.054	2.262	2.441
ITG-Grace2010	5.561	0.064	5.490	5.695

Table 6: Statistics of the 0.5°x0.5° local σ_ξ values relevant to the four models (units: arc-second)

Model	Mean	Standard deviation	Minimum	Maximum
EIGEN-CG01C	0.892	0.025	0.853	0.935
GGM03C	0.865	0.028	0.821	0.915
EGM2008 (400)	0.323	0.009	0.309	0.338
ITG-Grace2010	0.873	0.025	0.836	0.917

Table 7: Statistics of the 0.5°x0.5° local σ_η values relevant to the four models (units: arc-second)

Model	Mean	Standard deviation	Minimum	Maximum
EIGEN-CG01C	1.137	0.050	1.063	1.227
GGM03C	1.144	0.065	1.047	1.260
EGM2008 (400)	0.420	0.020	0.390	0.455
ITG-Grace2010	0.873	0.016	0.858	0.908

In order to compare the local uncertainties of the four studied harmonic models with the observational noise standard deviations, such uncertainties were evaluated at the scattered locations of some gravitational observations having the five types. Tables 8 to 12 list the corresponding statistical comparisons.

Table 8: Comparison among the scattered geoidal height data noise and the co-located models' σN values (units: m)

Model	Mean	Standard deviation	Minimum	Maximum
N/P (data)	0.031	0.018	0.001	0.050
EIGEN-CG01C	0.233	0.003	0.228	0.237
GGM03C	0.175	0.005	0.167	0.180
EGM2008 (400)	0.091	0.001	0.089	0.092
ITG-Grace2010	0.209	0.002	0.206	0.212

Table 9: Comparison among the scattered gravity anomaly data noise and the co-located models' $\sigma_{\Delta g}$ values (units: mgal)

Model	Mean	Standard deviation	Minimum	Maximum
N/P (data)	0.723	0.394	0.009	1.000
EIGEN-CG01C	6.351	0.098	6.129	6.563
GGM03C	6.307	0.142	5.984	6.605
EGM2008 (400)	2.324	0.039	2.235	2.407
ITG-Grace2010	5.494	0.050	5.427	5.615

Table 10: Comparison among the scattered gravity disturbance data noise and the co-located models' $\sigma_{\delta g}$ values (units: mgal)

Model	Mean	Standard deviation	Minimum	Maximum
N/P (data)	0.017	0.006	0.009	0.022
EIGEN-CG01C	6.250	0.028	6.200	6.280
GGM03C	6.115	0.041	6.042	6.157
EGM2008 (400)	2.283	0.011	2.262	2.295
ITG-Grace2010	5.495	0.003	5.490	5.500

Table 11: Comparison among the scattered meridian vertical deflection data noise and the co-located models' σ_ξ values (units: arc-second)

Model	Mean	Standard deviation	Minimum	Maximum
N/P (data)	1.600	0.000	1.600	1.600
EIGEN-CG01C	0.903	0.025	0.854	0.931
GGM03C	0.879	0.028	0.822	0.910
EGM2008 (400)	0.327	0.009	0.309	0.337
ITG-Grace2010	0.885	0.025	0.837	0.913

Table 12: Comparison among the scattered prime-vertical deflection data noise and the co-located models' σ_η values (units: arc-second)

Model	Mean	Standard deviation	Minimum	Maximum
N/P (data)	1.600	0.000	1.600	1.600
EIGEN-CG01C	1.182	0.053	1.065	1.215
GGM03C	1.203	0.069	1.049	1.246
EGM2008 (400)	0.437	0.021	0.391	0.450
ITG-Grace2010	0.890	0.015	0.859	0.902

4. Conclusions

From Table 3 to 7, it could be concluded that the local commission errors of each geopotential models are significant and tend to be uniform over the investigated geographical window. Among such models, the EGM2008 model, up to degree 400, possesses a superior local precision values, regarding the five considered anomalous features. This result, which is associated with the highest locally investigated resolution in the current work, represents a good check on the numerical stability and convergence of the mathematical algorithm followed in Eqs. (17) to (21).

Regarding geoidal height uncertainties, the GGM03C model has a considerable improvement over EIGEN-CG01C. Slight improvements exist, regarding the other gravitational quantities.

In spite of being a satellite-only harmonic model, ITG-Grace2010 has also remarkably better local undulation precisions than that of the combined EIGEN-CG01C model, but is still worse than GGM03C. However, these three models show almost equal local uncertainties of the meridian deflection component. Considering the gravity anomaly, gravity disturbance and prime-vertical deflection components, the ITG-Grace2010 seems slightly better than the other two models.

Comparing Tables 3 to 7 with Table 2, one could conclude that for each model, the resulting local commission errors are typically greater than the global ones. Such conclusion is valid for the five considered anomalous quantities.

Comparing Table 8 to 12 with Table 3 to 7, it is obvious that the local precisions over the grid nodes are close to those computed at the scattered data locations, both having very small standard deviations about the respective mean values. This ensures that regarding the study window, such local uncertainties are almost uniform for a geopotential model-derived quantity.

Finally, Table 8 to 12 show that the local uncertainties of the gravimetric quantities, to be derived from geopotential models, are much larger than the relevant observational noise.

5. Recommendations

Based on the above conclusions, it is recommended to take into account such significant local uncertainties of geopotential models during the evaluation of geopotential harmonic models, in regard to different observed gravitational data. Also, the local noise of geopotential models should be incorporated in the remove-restore technique while solving for local geoid, anomaly or deflection models via the various integral methods. Finally, the algorithm that was used in the current work could be further examined, regarding its capability of locally flagging the maximum degree, behind which the coefficients' signal-to-noise dramatically degrades.

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