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Use of heuristic methods for finding the condition that satisfies the desired precision criteria at minimal cost

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Abstract: Several heuristic methods have evolved in the last decade that facilitate solving optimization problems that were previously difficult or impossible to solve. Deformation monitoring is a kind of continuous recording positions (horizontal and vertical coordinates) precisely regardless the deformation pattern and instrument used. In general, a deformation monitoring network can be designed using the trial and error method or analytical methods such as linear programming and nonlinear programming. Recently, deformation monitoring networks have been designed by heuristic optimization algorithms such as Genetic Algorithms (GAs), Particle Swarm Optimization (PSO) and Simulated Annealing (SA). In this paper, GAs and PSO are applied to a geodetic horizontal deformation monitoring network to solve second-order design problem. The results proved that both GAs and PSO can be used as alternative methods in place of the traditional optimization techniques with high efficiency.

Keywords: Deformation monitoring networks; Heuristic optimization techniques; Second Order Design (SOD); Genetic algorithms; Particle swarm optimization.

1. Introduction

Whenever any stress is applied to an object or a surface, the object or surface might be prone to changes in its shape and form, also known as deformations (e.g. elongation, compression or distortion). Any object, natural or man-made, undergoes changes in space and time. It is very important to ensure stability of engineering structures as it is related to safety and life. Therefore, recently deformation analysis has gained more attention.

There are several techniques for measuring the deformations. These can be grouped mainly into two as: geodetic survey, which include conventional (terrestrial such as precise leveling measurements, angle and distance measurements etc.), photogrammetric (terrestrial, aerial and digital photogrammetry), satellite (such as Global Positioning System-GPS, InSAR), and non-geodetic techniques using lasers, tiltmeters, strainmeters, extensometers, joint-meters, plumb lines, micrometers etc. The emphasis of the present study is on geodetic methods.

One of the main aims of geodesy is detection of the deformations imposed on an object or an area which is characterized with points of a geodetic network. Since it is essential to detect deformations for many purposes (monitoring plate tectonics, determination of global datum, taking precautions for a construction which may be under damage, etc.), considerable efforts and investigations have been performed on deformation analysis (Kavouras, 1982; Chen 1983; Chrzanowski et al. 1983).

Before any deformation measurement campaign is started, the geodesists should know about the result of their work according to the set objectives. This leads to the need for optimization and design of deformation monitoring schemes. Essentially, the purpose for the optimization and design of monitoring schemes is to prevent the deformation measurement campaigns from failing. It enables one to make decisions on which instruments should be selected from the hundreds of available models and where they should be located in order to estimate the unknown parameters and achieve the desired criteria derived from and determined by the purpose of the monitoring scheme (Kuang, 1996).

Following the convention of design orders for geodetic networks by Grafarend, (1974), one may consider the same classification of the design orders for deformation monitoring networks. There are, however, significant differences in the design problems in positioning networks versus monitoring networks (Kuang, 1991). The classification of the optimization problems (design orders):

a) **Zero Order Design (ZOD):** It is the search for an optimal datum. But here in the deformation monitoring network there is no ZOD problem (Chen and Chrzanowski, 1986; Kuang, 1991).

b) **First Order Design (FOD)**: It involves the geometric shape of the network including the optimum number and location of the geodetic stations.

c) **Second Order Design (SOD):** It deals with the determination of the weights of network measurements.

d) **Third Order Design (THOD) Problem:** Improvement of existing networks might be very useful for monitoring networks.

Analytical techniques, such as linear and nonlinear programming, have been used for geodetic optimization tasks. On the other hand, some heuristic optimization techniques have been explored recently in geodetic science such as genetic algorithms (GA), simulated annealing (SA) and particle swarm optimization (PSO) algorithms (Saleh and Chelouah, 2004; Sahabi et al., 2008; Yetkin et al., 2008, 2009 and 2011; Dwivedi and Dikshit, 2013; Doma, 2013).

The major motivation of this study as the subject of this paper is to solve the SOD problem using heuristic techniques and make a comparison between the results of using these techniques and the previous results of using analytical method for the same deformation network as in Kuang (Kuang, 1991).

2. Observing campaigns

If the geodetic observables involved in a campaign can be considered in a network without a configuration defect, then the vector of observations, can be related to the unknown coordinates, x, of the points or stations involved by:

$$\boldsymbol{L} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{v} \quad (1)$$

where
$\underline{\boldsymbol{L}}$ is an n-vector of observations, $\underline{\boldsymbol{x}}$ is a u-vector of unknown parameters,
$\underline{\mathbf{v}}$ is an error vector and $\underline{\boldsymbol{A}}$ is the design matrix.

The least squares estimates of the coordinates are obtained by (Wells and Krakiwsky, 1971; Vanicek and Krakiwsky, 1986; Amiri-Simkooei et al., 2012)

$$\mathbf{x} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{P}\,\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{P}\,\mathbf{L} \quad (2)$$

Here **P** is the weight matrix of the observables, the inverse of their covariance, **C.** The variance-covariance matrix $\mathbf{C}_{\mathbf{x}} = \sigma_{\mathbf{o}}^{2}\left(\mathbf{A}^{\mathrm{T}}\mathbf{P}\,\mathbf{A}\right)^{-1}$ provides the knowledge of the accuracy of the coordinates corresponding to the combination of the choice of instrumentation and observation techniques, through the matrix **P,** and of the configuration of the network, through **A.** In most instances, $\sigma_{\mathbf{o}}^{2}$ (variance factor known *a priori*) is taken as unity. In an actual adjustment, $\boldsymbol{L}$ in Equation (2) is the misclosure vector $\mathbf{w} = \mathbf{Ax} - \boldsymbol{L}$ since the normal equations are non-linear but this is not of consequence in the design or pre-analysis.

The design for deformation monitoring assumes that the same configuration and observables will be involved in the repetition of a campaign. Consequently, the process can be extended to consider a pair of campaigns. The deformation can be described, in a displacement field, dx, as the difference in coordinates between the two campaigns, i.e., $\boldsymbol{dx} = \boldsymbol{x}_2 - \boldsymbol{x}_1$, the covariance matrix of displacement component is $\boldsymbol{C}_{dx} = \boldsymbol{C}_{x1} + \boldsymbol{C}_{x2}$, so the weight matrix is $\boldsymbol{P}_{dx} = \boldsymbol{C}_{dx}^{-1}$, and campaign 2 following campaign 1. This displacement field would be the "observed" displacement field since it results from measurements and its displacement components are located only at points involved in the network of observables. The observed displacement field is related to the deformation model parameters, ***c***, through (Secord, 1995):

$$\mathbf{dx} + \mathbf{v} = \mathbf{Bc} \quad (3)$$

by the modeling design matrix **B**. The least squares estimates of the deformation parameters are then obtained from (Kuang, 1991; Yetkin et al., 2009):

$$\mathbf{c} = \left(\mathbf{B}^{\mathrm{T}}\mathbf{P}_{\mathbf{dx}}\mathbf{B}\right)^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P}_{\mathbf{dx}}\mathbf{dx} \quad (4)$$

with the covariance matrix of the parameters, $\mathbf{c}_{\mathbf{c}} = \left(\mathbf{B}^{\mathrm{T}}\mathbf{P}_{\mathbf{dx}}\mathbf{B}\right)^{-1}$.

For design purposes, the covariance of the deformation parameters can be related directly to the covariance of the observables by combining the above to yield:

$$\boldsymbol{c}_{c} = 2\,.\left(\boldsymbol{B}^{T}\boldsymbol{A}^{T}\boldsymbol{C}^{-1}\boldsymbol{A}\,\boldsymbol{B}\right)^{-1} \quad (5)$$

By specifying the type of instrumentation and the observation techniques, the elements of $\boldsymbol{C}^{-1}$ are defined ($\boldsymbol{C}^{-1} = \boldsymbol{P}$ in Equation (2)).

Criterion matrices are adequate tools to set up objective function (Doma, 2014). Let us consider the case in which a criterion matrix $\boldsymbol{C}_{c}^{c}$ for deformation parameters has been chosen as the precision criterion, the design problem then seeks an optimal weights such that it can be best approximated by C_c (Kuang, 1991 and 1996; Yetkin et al., 2008 and 2011; Baselga, 2011)

$$\min\sqrt{\sum_{\mathbf{i}}\sum_{\mathbf{j}}\left((\mathbf{C}_{\mathbf{c}}^{\mathbf{c}})_{\mathbf{ij}} - (\mathbf{C}_{\mathbf{c}})_{\mathbf{ij}}\right)^{2}} \quad (6)$$

This approach to design can guide in selecting the instrumentation, the techniques of observation, the location of the points and the deformation model. In general, there are several techniques that can be applied for solving this problem. The two main kinds of techniques are analytical optimization techniques and heuristic optimization techniques.

3. Heuristic optimization algorithms

The word "heuristic" is used to describe algorithms that are effective at solving complex problems quickly. In such problems the objective is to find the optimal solution. i.e. one that minimizes or maximizes an objective function. In the present case, the objective function is given in Eq. 6 and aim is to minimize it. Recently, optimization problems for deformation monitoring networks have been solved by heuristic optimization techniques such as Evolutionary algorithms (EAs), PSO and SA. A basic strategy for a heuristic as applied to design the monitoring networks could be as follows:

a) Choose initial parameters,
b) Swap two of the objective functions to make the objective of a lower value
c) Repeat step 2 until no improvements can be made.

The goal of this study is solving the mathematical model in Equation (6) for a pre-solved example in Kuang (Kuang, 1991) using GAs and PSO, and compare these results with the results of Kuang (Kuang, 1991) who used analytic technique.

4.1 Genetic algorithms (GAs)

GAs are effective searching methods in a very wide and huge space. In GAs, the design space must be changed into the genetic representation (Goldberg, 1989). Therefore, GAs deal with a series of encoded variables. The advantage of using encoded variables is that it is possible to encode continuous functions like discrete functions. GAs are based on random processing or more specifically it is based on guided random process. Therefore, random operators of searching space are examined in a comparative way.

Basically, in order to use GAs the following three concepts must be defined:

a) ***Objective function*****:** In each problem, the purpose is to maximize or minimize a parameter or parameters. Therefore the objective function is determined using mathematical relations and proper weighing to solve the problem.

b) ***Searching space:*** The purpose of problem solving is to find the best result among different results. The space of all probable states is called searching space. Each result could be represented by a value which determines its propriety.

c) ***Operators of GAs*****:** After achieving the objective function and encoding the population, it is the time for operators of GAs to start functioning. In the simple GAs, the three main operators, namely reproduction, merging and mutation, are usually used. Deformation monitoring networks can be optimized using genetic algorithms (Sahabi et al., 2008; Doma and Elshouny, 2011).

4.1.1 Genetic algorithms technique

The workability of genetic algorithms (GAs) is based on Darwinian's theory of survival of the fittest. Algorithmically, the basic genetic algorithms (GAs) are outlined as below (Sivanandam and Deepa, 2008):

Step (I) Start: Generate random population of chromosomes, that is, suitable solutions for the problem. Population being combination of various chromosomes is represented as in Figure 1. Thus the population consists of four chromosomes (Sivanandam and Deepa, 2008).

Step (II) Fitness: Evaluate the fitness of each chromosome in the population.

Step (III) New population: Create a new population by repeating following steps until the new population is complete:

Selection: Select two parent chromosomes from a population according to their fitness. Better the fitness, the greater the chance to be selected as parent.

Population	Chromosome 1	1 1 1 0 0 0 1 0
	Chromosome 2	0 1 1 1 1 0 1 1
	Chromosome 3	1 0 1 0 1 0 1 0
	Chromosome 4	1 1 0 0 1 1 0 0

Figure 1: Population

Crossover : With a crossover probability, cross over the parents to form new offspring, that is, children. If no crossover was performed, offspring is the exact copy of parents.

Mutation: With a mutation probability, mutate new offspring at each locus. The mutation process shown in Figure 2 is very simple.

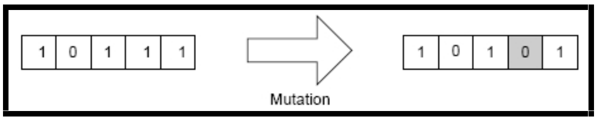

Figure 2: Illustration of Mutation Operator

Elitism [Accepting]: Place new offspring in the new population

Step IV ***Replace:*** Use new generated population for a further run of the algorithm.

Step V Test: If the end condition is satisfied, stop, and return the best solution in current population.

Step VI Loop: Go to step II.

Finally, the chromosome that has highest fitness is chosen as the optimized solution.

MATLAB ver. 7 has a function for optimization using GAs technique.

4.2 Particle Swarm Optimization (PSO)

PSO was originally designed and introduced by Eberhart and Kennedy in 1995 based on social intelligence of a group of birds or fishes (Kennedy and Eberhart, 1995 and 2001; Shi and Eberhart, 1998). Compared with other optimization algorithms, the PSO is more objective and easy to perform. It is applied in many fields such as the function optimization, the neural network training, the fuzzy system control, etc. In PSO algorithm, each individual is called "particle", which represents a potential solution. The algorithm achieves the best solution by the variability of some particles in the tracing space. The particles search in the solution space following the best particle by changing their positions and the fitness frequently, the flying direction and velocity are determined by the objective function.

In binary PSO, a population (swarm) of birds (possible solutions or individuals or particles) is initialized randomly with values of {0, 1}. It means each particle is a combination of one and zero which indicate the presence or absence of corresponding coefficient in the cost function respectively. These particles are represented as the current positions *(p)*. Then the fitness values of these particles are calculated using the cost function (Equation 6). Based on these fitness scores, the best positions of each particle ***(PBest)*** and the global best position of all particles ***(GBest)*** are determined. In an iterative process, the velocity of each particle *(v)* is updated as below (Yavari et al., 2012):

$$v_{ij}(t+1) = w(t).\ v_{ij}(t)+C_1.\ r_1.[GBest_i\ (t)-P_{ij}(t)]+C_2.r_2.[PBest_i\ (t)-P_{ij}(t)] \quad (7)$$

where,
i is the index of particle in the population;
j: is the index of bits in the binary string of each particle;
t is the iteration number;
r_1 and r_2 are two uniform random values in [0,1];
C_1 and C_2 are two constant acceleration coefficients and
w(t) is time varying inertia weight.

A nonlinear inertia weight (w) is used to adjust the effect of the current velocities in computation of the new velocity values as:

$$w(t) = w_{min} + (w_{max} - w_{min}).\ \frac{t_{max} - t}{t} \quad (8)$$

where,

w_{max} *and* w_{min} are two constant experimental parameters, and
t_{max} is the maximum number of iterations.
Once the velocity for each particle is calculated, each particle's position is updated by applying the new velocity to the particle's previous position:

$$x_i\ (t + 1) = x_i\ (t) + v_{ij}\ (t + 1) \quad (9)$$

The three steps of velocity update, position update and fitness calculations are repeated until a desired convergence criterion is met.

Currently, several studies are being carried out in the area of particle swarm optimization and hence the application area also increases tremendously (Sivanandam and Deepa, 2008; Yetkin et al., 2008, 2009 and 2011; Doma and Sedeek, 2012; Doma, 2013; Dwivedi and Dikshit, 2013).

5. Applied case study

As shown in Fig. 3, the network consists of 6 points is taken from Kuang (1991). The simulated approximate coordinates of all the network points are given in Table 1.

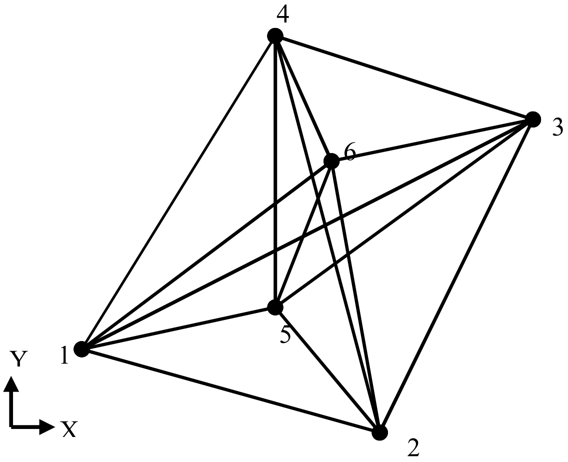

Figure 3: The horizontal monitoring network

Assume that the deformation model to be detected includes a homogeneous strain field over the whole area plus single point movements of points # 3, # 4 and # 5. That is, the vector of deformation parameters to be detected can be expressed as:

$$\mathbf{e} = (d_{x_3}\ d_{y_3}\ d_{x_4}\ d_{y_4}\ d_{x_5}\ d_{y_5}\ \varepsilon_x\ \varepsilon_{xy}\ \varepsilon_y)^T \quad (10)$$

where dx_i, dy_i (i=3, 4, 5) represent the displacements of points # 3, # 4 and # 5 in x-and y-directions respectively, and ε_x, ε_y and ε_{xy} the normal strain and shear strain parameters respectively. The deformation model can be expressed as:

$$\left.\begin{aligned} u_i &= \varepsilon_x x_i + \varepsilon_{xy} y_i \\ v_i &= \varepsilon_{xy} x_i + \varepsilon_y y_i \end{aligned}\right\} \quad \text{for } i = 1, 2, 6 \text{ and} \quad (11)$$

$$\left.\begin{aligned} u_j &= dx_j + \varepsilon_x x_j + \varepsilon_{xy} y_j \\ v_j &= dy_j + \varepsilon_{xy} x_j + \varepsilon_y y_j \end{aligned}\right\} \quad \text{for } j = 3, 4, 5 \quad (12)$$

We assume that the displacements have to be determined with a standard deviation of 0.71 mm while the strains with a standard deviation of 0.14 ppm. The following diagonal matrix will be used as the precision criterion matrix, i.e.,

$$C_e = 2\ .\ \text{Diag}\ [(0.5\ \text{mm})^2, \ldots, (0.5\ \text{mm})^2, (0.1\ \text{ppm})^2, \ldots, (0.1\ \text{ppm})^2] \quad (13)$$

The target function for precision is then used to best approximate the above criterion matrix is equation (6).
As for the initial observing plan, we assume to use only an EDM instrument to measure all the possible distances.

To achieve the above set design criteria, an observing plan has to be determined. To determine an optimum observing plan, we assume that we can have a choice of an EDM instrument with accuracies ranging from

$\sigma_s^2 = (1\text{mm})^2 + (1\,\text{ppm}.S_{km})^2$ to $\sigma_s^2 = (0.1\,\text{ppm}.S_{km})^2$,

where S_{km} is the distance computed from the approximate coordinates. This model aims to best approximation of the given precision criterion matrix.

Table 1: The simulated approximate coordinates of network points

Point	Approximate coordinates	
	X(m)	Y(m)
1	1125	1625
2	4625	375
3	6250	4625
4	3250	5875
5	3375	1500
6	4375	4625

The PSO parameters used in this research are shown in Table 2. These parameters are selected based on criteria given by Yetkin et al. (2011) and also experimentally to balance the global and local search of PSO. However, it should be noticed that PSO is rather stable to the mild changes of these parameters. The GAs parameters are set as listed in Table -3. We choose "Chromosomes length= 32bit" because we expect our number to be two integer digit and six decimal digits.

Table 2: PSO parameters

Parameter	Value
No. of particles	30
Iteration	300
(C1)	1.75
(C2)	1.1

Table 3: GAs parameters

Parameter	Value
Generations	500
Population size	100
Chromosomes length	32 bit

After the optimization solution process is done by using heuristic techniques (both PSO and GAs), the optimization results obtained from optimization model are listed in Tables 4 and 5.

From Table 4 we can see the Goodness of fitting of the precision criteria for the analytical technique (from Kuang, 1991), PSOadf5 technique and GAs technique, the precision criteria are less than and close to the required value for all used techniques.

Table 5 lists the initial weights, optimal weights using the analytical method (Kuang, 1991) and both PSO and GAs techniques. From this table one can see that, the sum of optimal weights which had been obtained from the analytical method is 69.926, but, the sum of weights which are obtained from heuristic techniques (both PSO and GAs) are 66.381 and 65.242 respectively. This means that the heuristic techniques can be used an alternative method to design a deformation network with high efficiency. Figure - 4 shows the performance of the heuristic techniques VS traditional techniques. From these results, one can see that summation of optimized weights for observed lines of the proposed network, GAs yields the smallest value then PSO; as we know that there is a direct relation between the cost of observations and their weights consequently the less weight the most economical.

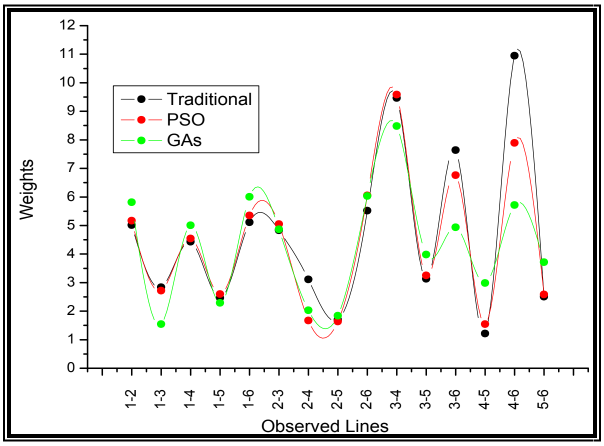

Figure (4): The performance of traditional, PSO and GAs techniques in designing deformation network.

Table 4: Goodness of fitting of the precision criteria for both the analytical method and heuristic optimization techniques (PSO and GAs)

Parameters	Required precision	Obtained Precision		
		Precision from analytical technique (Kuang, 1991)	Precision from heuristic optimization techniques	
			PSO	GAs
dx_3	0.71 mm	0.52 mm	0.53 mm	0.57 mm
dy_3	0.71 mm	0.71 mm	0.71 mm	0.71 mm
dx_4	0.71mm	0.66 mm	0.68 mm	0.71 mm
dy_4	0.71 mm	0.59 mm	0.60 mm	0.58 mm
dx_5	0.71 mm	0.68 mm	0.67 mm	0.67 mm
dy_5	0.71 mm	0.65 mm	0.63 mm	0.54 mm
ε_x	0.14 ppm	0.14 ppm	0.14 ppm	0.14 ppm
ε_{xy}	0.14 ppm	0.10 ppm	0.11 ppm	0.10 ppm
ε_y	0.14 ppm	0.11 ppm	0.11 ppm	0.11 ppm

6. Conclusions

The paper investigated use of two heuristic optimization techniques namely, PSO and GAs, to solve a second order deformation monitoring network. GAs and PSO were applied to a geodetic horizontal deformation monitoring network. The performance was compared with analytical methods. The results indicated that the heuristic optimization techniques have better efficiency than the analytical method in solving the SOD problem.

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Table 5: The distances and desired weights of the observations obtained by the analytical method and heuristic techniques (both PSO and GAs)

Observations		Distance (meter)	Initial weights (P_i)	Optimal weights		
				Analytical method (Kuang, 1991)	Heuristic techniques	
From	To				PSO	GAs
1	2	3716.52	0.0675	5.0164	5.1654	5.8146
1	3	5938.49	0.0276	2.8356	2.7168	1.5440
1	4	4751.65	0.0424	4.4291	4.5423	5.0043
1	5	2253.47	0.1645	2.4641	2.5961	2.2887
1	6	4422.95	0.0486	5.1118	5.3483	6.0010
2	3	4550.07	0.0461	4.8302	5.0461	4.8674
2	4	5669.27	0.0302	3.1113	1.6716	2.0321
2	5	1681.70	0.2612	1.7022	1.6334	1.8381
2	6	4257.35	0.0523	5.5172	6.0516	6.0323
3	4	3250.00	0.0865	9.4675	9.5852	8.4820
3	5	4246.32	0.0526	3.1334	3.2513	3.9821
3	6	1875.00	0.2215	7.6396	6.7535	4.9390
4	5	4376.79	0.0496	1.2132	1.5445	2.9852
4	6	1681.70	0.2612	10.9482	7.8913	5.7141
5	6	3281.10	0.0850	2.5063	2.5833	3.7176
Summations		-------	1.4968	69.9261	66.3807	65.2425