Journal of Geomatics

Vol 8 No. 1 April 2014

Orthometric correction for trigonometric leveled heights

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(Received: Jun 04, 2013; in final form Jan 27, 2014)

Abstract: In this study, a proposed computational scheme is applied for the assessment of the orthometric correction for long line trigonometrically leveled height differences. This algorithm is based on the spherical harmonic coefficients of geopotential models and trigonometric elevation data. The applied algorithm does not demand any terrestrial gravity data and is route independent. In particular, two geopotential models with different resolutions were utilized. The results showed a reasonable applicability of the investigated algorithm to compute the orthometric correction for trigonometric leveling. Thus it is recommended to use this approach for computing the orthometric corrections in similar modern heighting applications, such as precise EDM trigonometric height traverses.

Keywords: Trigonometric leveling**,** Height traverses, Orthometric correction, Geopotential models

1. Introduction

Due to the non-parallelism of level surfaces, precise leveling requires the application of the orthometric correction (OC) to obtain the required orthometric height (OH). Being a correction for a systematic error, the OC must be accounted for prior to the adjustment of spirit leveling networks (Featherstone and Kuhn, 2006). The OC could be rigorously assessed, via the combination of observed gravity values along spirit leveling routes (Hwang and Hsiao, 2003). In spirit leveling networks, reciprocal trigonometric height differences may be used over small distances (< 1 km) to fill eventual small gaps (Hofmann-Wellenhof and Moritz, 2005).

The method of trigonometric heighting is still commonly used. The precision of trigonometric height traverses could be increased (to a few mm/√km) via the reciprocal or leap-frog observational mode. Also, by extending the lengths of sight to a few hundred metres, the number of set-ups per km is minimized, which is a significant source of error in spirit leveling. So, short length trigonometric leveling could be an efficient alternative to spirit leveling in hilly regions (Chrzanowski, 1989). It could be more efficient than GPS/Leveling (Ceylan et al., 2005). Also, such heighting technique can be used for monitoring the vertical deformations (Kovačič and Kamnik, 2007).

For trigonometric heighting, when long lines (a few km or more) are encountered, simultaneous reciprocal observations must be used, so that the effect of atmospheric refraction could be mathematically eliminated and independently checked (Kharaghani, 1987).

Long line trigonometric leveling was the unique tool for translating elevations for terrestrial triangulation stations, which were often mounted over hilly locations (Heiskanen and Moritz, 1967). Today, it may be used for special applications, e.g., for the height determination of inaccessible sites (Torge, 2001). Nevertheless, long line triangulated heights could be of benefit for relevant datum transformations. So, in general, an accurate knowledge of trigonometric elevations may be sought.

Regarding spirit levelling, a computational algorithm was proposed in a previous work, in which the OC could be expanded in spherical harmonics. This algorithm was based on the global geopotential coefficients and spirit leveled height data. In such algorithm, the effect of the Earth's rotation on OC was found to be negligible (Hassouna, 2013). The algorithm does not need any terrestrial gravity data and is so general, that it can work independent on any route. So, such algorithm could be suitable for computing the OC for trigonometric leveled height differences.

Motivated from the above, the objective of the current work is to extend applying the above algorithm for investigating the OC of long line trigonometric elevation differences. The investigated procedure could also be applicable to (short line) trigonometric height traverses.
Namely, some existing observed trigonometric elevations and elevation differences between station pairs are considered. These stations lie along the Nile Valley and have known geodetic coordinates with respect to the WGS-84 reference ellipsoid. For this purpose, beside the trigonometric height data, the global geopotential models GGM03C (Tapley et al., 2007) and GOCO03S (Mayer-Gürr et al., 2012) are used. The GGM03C model is a combined model, up to degree and order 360, whereas GOCO03S is a satellite-only model with a maximum degree of 250.

2. Geometry of trigonometric leveling

Figure 1 shows a simple geometric scheme of a one way trigonometric leveling between two terrain points A and B. Utilizing the simultaneous reciprocal procedure, the combined systematic effect of refraction and the Earth's curvature could be mathematically eliminated. In this case, the trigonometrically leveled height difference may be computed as follows

$$\Delta H_{AB} = H_B - H_A = [(hi_A + hr_A) - (hi_B + hr_B) + d\,(\cos Z_A - \cos Z_B)]/2, \quad (1)$$

with

H_A	the leveled elevation of A,
H_B	the leveled elevation of B,
hi_A & hi_B	the heights of instrument at A and B,
hr_A & hr_B	the heights of target at A and B,
Z_A & Z_B	the observed zenith angles at A and B,
d	the observed slant distance AB.

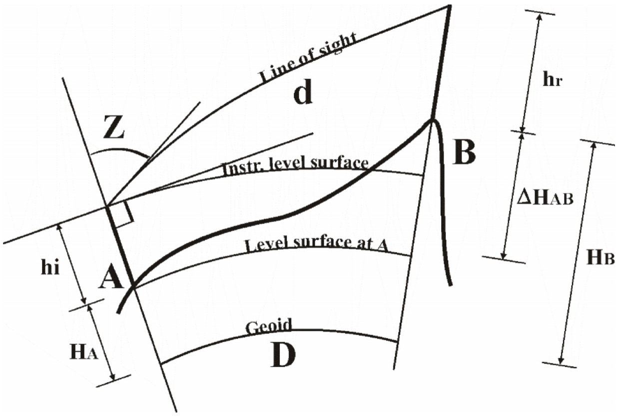

Figure 1: Scheme of trigonometric leveling

The precision of simultaneously reciprocal observed height differences could be about 8 mm /km. So, a cm-order precision of long line reciprocally observed height differences can be achieved over distances of a few km, whereas a dm-order uncertainty is expected with larger distances (Torge, 2001; Hwang and Hwang, 2002). Equation (1) is based on the simple assumption that the geoid and level surfaces are concentric spheres, as depicted in Figure 1.

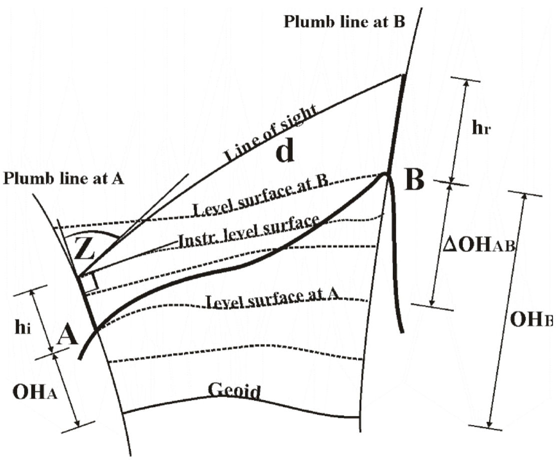

Figure 2: Realistic geometry of trigonometric leveling

Being related to the local plumb lines, trigonometrically leveled heights also suffer from the convergence of the non-parallel level surfaces at the instrument and reflector stations. This is true for either short or long distance trigonometric leveling. Figure 2 shows the geometry of trigonometric leveling, but depicting the actual irregular shape of the non-parallel level surfaces, including the geoid.

Obviously, one should state here the following definitions

OH_A the orthometric height of A, measured along its plumb line from the geoid to its level surface,
OH_B the orthometric height of B, measured along its plumb line from the geoid to its level surface,
ΔOH_{AB} the orthometric height difference, measured along the plumb line of the station B, from the level surface of A to B,

$$OC_A = OH_A - H_A, \quad (2a)$$
$$OC_B = OH_B - H_B, \quad (2b)$$
$$OC_{AB} = \Delta OH_{AB} - \Delta H_{AB}, \quad (2c)$$

where
OC_A is the orthometric correction to the elevation of A,
OC_B is the orthometric correction to the elevation of B.

Generally speaking, for a leveling profile in the Alps region, OC_{AB} may amount to 15 mm per 100 m of measured height difference (Hofmann-Wellenhof and Moritz, 2005). Hence, for large trigonometric height differences, the magnitude of OC_{AB} could be significant. So, applying such correction to the relevant height differences increases their accuracy. Again, being a systematic effect, the magnitude of OC_{AB} should not be compared with that of a height difference uncertainty or random parts of loop closures (Hwang and Hsiao, 2003). This means that the magnitude of such systematic effect must be applied, even if it is smaller than the standard error of the observed height difference (Allister and Featherstone, 2001).

3. Algorithm

Neglecting the minute effect of the Earth's rotation, the OC of the leveled elevation of a terrain station could be looked upon as composed from the modulation of the topographic signal (elevations) on the gravitational signal. As the gravitational signal can be expressed in terms of the (unitless) geopotential harmonic coefficients, thus beginning with harmonic degree two, the OC may be computed as follows (Hassouna, 2013)

$$OC = OH - H = H \sum_{n=2}^{L} \sum_{m=0}^{n} [(\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda)\, \bar{P}_{nm}(\cos\theta)], \quad (3)$$

where

- L the maximum degree of the used geopotential harmonic model,
- θ the co-latitude,
- λ the geodetic longitude,
- $\bar{C}_{nm}$ the fully normalized spherical harmonic C-coefficients of degree n and order m,

$\bar{S}_{nm}$ the fully normalized spherical harmonic S-coefficients of degree n and order m,

$\bar{P}_{nm}(\cos\theta)$ the fully normalized associated Legendre function of degree n and order m.

Similar to Eq. (3), the orthometric correction, OC_{AB}, for the trigonometric elevation difference between two stations A and B, could be expressed as (Hassouna, 2013)

$$OC_{AB} = \Delta OH_{AB} - \Delta H_{AB}$$

$$= \Delta H_{AB} \sum_{n=2}^{L} \sum_{m=0}^{n} [(\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda)\, \bar{P}_{nm}(\cos\theta)], \quad (4)$$

where

θ_B & λ_B the co-latitude and longitude, respectively, of station B.

The OC_{AB} could also be expressed as the difference between the OCs of the elevations of A and B, as follows

$$OC_{AB} = OC_B - OC_A, \quad (5)$$

where OC_A and OC_B can be separately evaluated via Eq. (3) at stations A and B, respectively.

4. Results

Table-1 lists the statistics of the trigonometric elevations, elevation differences and horizontal distances (D) between the pairs of stations under consideration. The large range of elevation differences is due to the fact that the majority of these points are mounted on hilly regions, whereas the other portion has moderate elevations.

Table 1: Statistics of the trigonometric leveled elevations, elevation differences and distances

	No.	Mean	Std. dev.	RMS	Min.	Max.
H (m)	85	269.93	130.24	102.62	37.81	695.89
ΔH (m)	84	-3.57	133.08	94.14	-605.55	312.22
D (km)	84	23	14	11	4	57

Firstly, Eq. (3) was used to compute the OCs for the individual station heights. Then, the differences among these OCs were computed to express the height difference corrections, OC_{AB}, for the relevant point pairs, according to Eq. (5). Finally, Eq. (4) was used to obtain the same features. Table-2 summarizes the statistics of the three computed sets of OC, using the GOCO03S harmonic model. Also, Table-3 lists the same results, as obtained from the GGM03C model. Moreover, the two tables show the statistics of the differences, ΔOC_{AB}, among the two latter sets as computed from Eqs. (4) and (5).

Finally, Table-4 illustrates the statistics of the ratios of the OC_{AB}, based on Eq. 4 and GGM03C, to the relevant height differences between station pairs. It also shows the statistics of the ratios of the absolute values of the OC_{AB} to the relevant horizontal distances, D. These distances were computed from the stations geodetic coordinates, using a mean Earth's radius of 6371 km. Such spherical approximation of the Earth's shape was found to be accurate enough for that purpose.

Table 2: Statistics of the three sets of OCs based on GOCO03S (mm)

	Mean	Mean (abs.)	Std. dev.	Min.	Max.
OC (Eq. 3)	66.8	66.8	36.5	6.2	171.3
OC_{AB} (Eqs. 5)	-1.2	22.6	32.8	-149.4	86.2
OC_{AB} (Eq. 4)	-0.7	22.5	32.6	-146.9	86.5
ΔOC_{AB} (Eq. 4 – Eq. 5)	0.5	0.5	0.5	0.0	2.3

Table 3: Statistics of the three sets of OCs based on GGM03C (mm)

	Mean	Mean (abs.)	Std. dev.	Min.	Max.
OC (Eq. 3)	66.8	66.8	36.5	6.2	171.4
OC_{AB} (Eqs. 5)	-1.2	22.6	32.8	-149.5	86.3
OC_{AB} (Eq. 4)	-0.7	22.5	32.6	-146.9	86.5
ΔOC_{AB} (Eq. 4 – Eq. 5)	0.5	0.5	0.5	0.0	2.6

Table 4: Statistics of the ratios of OC_{AB} to both the height difference and horizontal distance based on GGM03C and Eq. 4

	Mean	Std. dev.	Min.	Max.
$OC_{AB}/\Delta H_{AB}$ (mm/100m)	23.9	4.4	16.5	31.5
$\lvert OC_{AB}\rvert /D_{AB}$ (mm/km)	1.2	1.4	0.0	7.7

5. Concluding remarks

From Tables-2 and 3, it may be concluded that the two geopotential models, although having different resolutions, resulted in almost the same values of OCs for trigonometric leveled elevations and elevation differences. This result agrees with the previous conclusion in Hassouna (2013), regarding spirit leveling. Namely, the OC, which expresses the linear convergence of level surfaces, may seemingly obey a relatively low frequency trend. In general, the resulting

OC magnitudes are significant and could improve any subsequent implementations of trigonometrically leveled heights. The last rows in Tables-2 and 3 imply that Eq. (4) may be equivalent to Eq. (5), regarding the resulting correction for trigonometric height differences. The average difference between the two methods is less than 1 mm.

The first row in Table-4 implies that the values of OC_{AB} are greater than that of the aforementioned Alps' leveling profile (of 15 mm per 100 m of height difference). Also, this table shows that the average OC_{AB} for the investigated height differences is 1.2 mm/km, a value which must not be ignored over spirit leveled lines. Namely, for spirit leveled lines, an OC_{AB} that is greater in magnitude than 0.2 mm/km should be taken into account (Torge, 2001).

Finally, based on the obtained results, it is recommended to apply the used algorithm for the assessment of OCs for trigonometrically leveled height differences. Such algorithm does not need any terrestrial gravity data and is route independent. So, it could be applied for similar modern applications such as precise EDM trigonometric height traverses. In such cases, it would be possible to correct height traverses closures for the convergence of level surfaces.

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