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Short Note

Assessing the quality of fuzzy land cover classification by similarity and certainty measures

M.S. Ganesh Prasad[1] and Manoj K. Arora[2]

[1]Department of Civil Engineering, The National Institute of Engineering, Mysore, India
[2] PEC University of Technology, Chandigarh, India
Email: ganeshprasad.nie@gmail.com, manoj.arora@gmail.com

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Abstract: Fuzzy approaches are being adopted for supervised digital classification of remote sensing images. However, the use of fuzzy classification methods is restricted when compared to hard classification methods in producing land use/land cover maps from remote sensing images. The major barrier for the wider adoption of fuzzy classifications is the difficulty in evaluating the classification accuracy, as the conventional measures of accuracy are not appropriate for such classifications. To overcome this barrier, many measures of soft classification accuracy have been developed. In this paper, two measures viz., fuzzy similarity measure and fuzzy certainty measure have been used for assessing the quality of a fuzzy classification . Fuzzy *c*- means classification was applied to a synthetic data set to derive fuzzy membership values. The derived fuzzy membership values and the corresponding fuzzy reference data were used to compute the values of fuzzy similarity measure and fuzzy certainty measure for each of the classes considered in the classification. The results indicated that the two measures estimate the values differently and fuzzy certainty measure resembles measure of goodness of fit used in statistical models.

Keywords: Fuzzy classification, Overall accuracy, Similarity, Fuzzy certainty, Goodness of fit

1. Introduction

Remote sensing from satellite based sensors provides synoptic views of the earth surface at regular time intervals, and has been considered as an attractive source of data acquisition. Transformation of observed remote sensing data in the form of spectral responses into thematic classes representing earth surface features is achieved by a number of image classification procedures. The use of fuzzy classification to produce accurate and reliable land cover maps is gaining importance due to its continuous nature of class representation. Fuzzy classification is a quantitative, iterative method for classifying thematic classes as continuous over geographic space. Fuzzy classification approaches aim to estimate the proportions of specific classes that occur within each pixel. The output of fuzzy or soft classifiers is a number of fraction images, one for each land cover class which describe the class composition. The fraction images representing 'soft' output may be derived using techniques such as fuzzy *c*-means clustering (Bezdek et al., 1984), linear mixture modeling (Settle and Drake, 1993), artificial neural networks (Foody, 1996) and possibilistic c-means clustering (Foody, 2000). Support vector machines (Brown et al., 2000; Varshney and Arora, 2004) have also been used to unmix the class composition within a pixel. Of these, fuzzy *c*-means algorithm is the one which has been used extensively for fuzzy classification in a range of applications.

Generally, the land use/ land cover map produced from remotely sensed data using the approaches mentioned above, may not exactly match the real ground situations due to various reasons including classification procedures, landscape characteristics, sensor resolution and spectral overlap. This discrepancy between the classified image and the real ground situation is a key concern for the utility of such data in many applications. For this reason, it is essential that the quality of image classification has to be assessed.

Quality of a fuzzy classification in terms of accuracy can be evaluated by the measures such as entropy (Maselli et al., 1994), Euclidean distance (Foody, 1996), L1 distance (Foody and Arora, 1996), cross entropy (Foody, 1995), measures of information closeness (Foody, 1996) and correlation coefficients (Maselli et al., 1996). The entropy as an accuracy measure is appropriate when the classification is fuzzy and the reference data are crisp (Shalan et al., 2003). The correlation coefficients are used to represent the accuracy of individual classes only. Higher values of correlation coefficient indicate better classification accuracy. Rest of the measures are useful when a fuzzy classification is evaluated with fuzzy reference data. However, the results obtained from these measures are generally not easy to interpret (Ricotta, 2004) as these are indicating the accuracy of classification indirectly. Lower the values of entropy/ cross entropy/ L1 distance and measures of information closeness, higher will be the accuracy.

Binaghi et al. (1999) proposed a method that used fuzzy set theory to extend the applicability of conventional error matrix method to evaluate fuzzy classification accuracy in the form of a fuzzy error matrix. The fuzzy error matrix was designed for those situations in which classification and/or reference data are expressed in multi membership form as well as crisp form. The formulae for fuzzy error matrix and accompanying accuracy measures depend on the type of sampling design used to collect the reference data and should be used carefully (Stehman et al., 2007). In spite of its sound theoretical basis, the fuzzy error matrix has generally not been accepted as a standard accuracy measure to report the accuracy of fuzzy classification (Silvan-Cardenas and Wang, 2008). Laba et al. (2002) utilised the concept of fuzzy operators (Gopal and Woodcock, 1994) to evaluate the accuracy of regional scale land cover maps produced from remote sensing data. The results showed that the assessment of fuzzy classification using fuzzy operators led to an improvement in map accuracy by about 19% to 23%.

A set of measures based on fuzzy similarity concept (Jager and Benz, 2000) have been used to evaluate accuracy of fuzzy classification and fuzzy reference data. The advantage of these measures is that they can be applied when either classification or reference data is hard. If both classification and reference data are hard, they often reduce to conventional error matrix based measures. Other approaches to evaluate the accuracy of fuzzy classification include fuzzy set based operators (Woodcock and Gopal, 2000), Renyi's generalized entropy function (Ricotta and Avena, 2002) and generalized Morisita's index (Ricotta, 2004), sub-pixel fractional error matrix (Latifovic and Olthof, 2004), probabilistic similarity index (Ricotta, 2004), concept of multi level agreement (Tran et al., 2005), cross comparison matrix (Pontius and Cheuk, 2006), fuzzy error matrix in the absence of ground data (Okeke and Karnieli, 2006), fuzzy certainty measure (Schiewe and Gahler, 2008) and sub-pixel confusion-uncertainty matrix (Silvan-Cardenas and Wang, 2008). A new family of overall accuracy, user's and producer's accuracies was also proposed by Gomez et al. (2008).

The main objective of this paper is to apply fuzzy certainty measure proposed by Schiewe and Gahler (2006) and a fuzzy similarity measure proposed by Jager and Benz (2000) to assess the quality of a fuzzy classification. A comparison of results obtained from these two measures is also made. Such a comparative evaluation of two measures may help in understanding their behaviour and relative efficacies. For the purpose, class membership values derived from fuzzy –c means classification of a synthetic image data is used. This paper is organized into four sections. The next section briefly discusses the measures of quality used in the present study. Section 3 describes fuzzy classification of experimental data and the result of classification. In Section 4, assessment of the quality of classification using two measures adopted has been provided. Finally, summary and conclusions are provided in Section 5.

2. Description of measures of quality

2.1 Measure of similarity

The main idea of fuzzy classification approach is to associate a pixel with every class considered in the classification scheme, with variable degree of class memberships. Partial class membership values derived from fuzzy classification can serve as baseline information to assess the quality of classification in terms of accuracy and uncertainties. Accuracy of a fuzzy classification can be assessed by constructing fuzzy error matrix and deriving the measures such as user's, producer's and overall accuracy (Binaghi *et al.,* 1999). Here, we provide the mathematical formulation for deriving the overall accuracy of a pixel in the context of fuzzy error matrix. The elements in the fuzzy error matrix are derived by using "MIN" operator introduced in the theory of fuzzy sets. Each value of $\mu(C)_{ji}$ is compared with the values of $\mu(R)_j$ for all i= 1,2,…N and the minimum of the two is assigned its corresponding position in the error matrix. The major diagonal elements $d(\mu_{ji})$ in the matrix that are used to estimate overall accuracy are computed as

$$d(\mu_{ji}) = \{ \mu(R)_{ji} \wedge \mu(C)_{ji} \} \quad \text{for } i = 1,2\ldots N \tag{1}$$

where, $\mu(R)_{ji}$ and $\mu(C)_{ji}$ are the proportions of i-th class in j-th pixel in fuzzy reference and fuzzy classification,

The 'SUM' of the major diagonal elements divided by the total grades of membership in the reference data/classification data, when the condition of orthogonality holds represents the overall accuracy (OA_{ji}) of that pixel (equation (2)). The overall accuracy thus obtained may be interpreted as a measure of the total match between the reference and classification data. This total match is dependent on 'MIN' operator used to obtain degree of matching for individual classes.

$$OA_{ji} = \sum_{i=1}^{N} \mu(R)_{ji} \wedge \mu(C)_{ji} \tag{2}$$

Other way of measuring the accuracy of classification is to measure the similarity between a fuzzy classification and fuzzy reference by considering the similarity between the corresponding fuzzy sets. This has been demonstrated by Jager and Benz (2000) by defining a fuzzy similarity measure which is a mapping s: $[0,1]^X \times [0,1]^X \rightarrow [0,1]$ assigning two fuzzy sets $R, C \in [0,1]^X$ a 'degree of similarity' $S(R,C) \in [0,1]$ subject to conditions

(*S1*) $S(R,R)=1$ for every fuzzy set R

(*S2*) $S(R,C)= S(C,R)$ for all fuzzy sets R,C

(*S3*) $S(R, D) \leq S(R,C) \wedge S(C, D)$ whenever $R \subset C \subset D$.

Here, condition (*S1*) states that if *R* and *C* are equal , there exists maximum similarity, (*S2*) states the symmetry and (*S3*) states that if *C* lies between *R* and *D* , then the degree of similarity of *R* to *D* is at most equal to both the degrees of similarity of *R* to *C* and *C* to *D*. And, for a fuzzy reference $R = \{ R_1 \ldots\ldots\ldots R_N \}$ and fuzzy classification $C = \{ C_1 \ldots\ldots\ldots C_N \}$, with a fuzzy similarity measure S, fuzzy overall accuracy $(FOA)_s$ is defined as

$$(FOA)_s = S\ ([C_1 \ldots\ldots\ldots C_N],\ [R_1 \ldots\ldots\ldots R_N]) \quad (3)$$

Although, four different measures of similarity were proposed by Jager and Benz(2000), we are providing the measure of similarity used in this study as,

$$(FOA)_{s1} = 1 - \frac{\sum_{i=1}^{N} \left| R_{ji} - C_{ji} \right|}{\sum_{i=1}^{N} (R_{ji} \vee C_{ji})} \quad (4)$$

2.2 Fuzzy certainty measure

To consider the influence of the reference data and to indicate the quality of classification procedure, Schiewe and Gahler (2006) proposed a new characteristic value, the Fuzzy Certainty Measure FCM(c) per class c as follows.

$$FCM(c) = 1 - \frac{1}{n} \sum_{i=1}^{i=n} \left| \mu_{i,REF}(c) - \mu_{i,CLASS}(c) \right| \quad (5)$$

$$\forall i \left| \mu_{i,REF} > 0 \vee \mu_{i,CLASS} > 0 \right.$$

with:

μ_{REF} (c): membership value of a pixel (or region) for class c in reference data
μ_{CLASS} (c): membership value of a pixel (or region) for class c in classification result
n: number of pixels (or regions) under consideration

The value of FCM(c) varies between 0 and 1. Higher values of FCM(c) indicate larger coincidence between the reference and the classification. In the present work, fuzzy similarity measure (Eq. 4) and fuzzy certainty measure (Eq. 5) have been adopted to assess the quality of a fuzzy classification.

3. Application example

In remote sensing image processing domain, to understand the factors contributing to classification errors and isolating them for further analysis through empirical studies, the use of real image data is needed. Such empirical studies might require the use of real data at different resolutions, varied spectral characteristics and sufficient proportion of mixed pixels with desired 'within class variability' and 'boundary effect'. However, availability of real data with desired characteristics and other difficulties may limit the scope of these studies. In such cases, it may be advantageous to use synthetic image data with desired and controlled spectral characteristics. In this study, a synthetic image resembling real remote sensing images has been used.

The mean and variance covariance values derived from a set of pure pixels in four different classes of a typical Landsat Enhanced Thematic Mapper Plus (ETM+) image have been adopted to generate synthetic images in six spectral bands assuming that each class follows normal distribution (Ganesh Prasad, 2015). The size of the synthetic images has been kept as 200 x 200 pixels for computational convenience. The class proportion vector for pixels in synthetic image is designed in such a way that these show varying degrees of class mixtures in different regions of the image. Figure 1 shows the proportions used to mix the spectral responses of four hypothetical classes of interest.

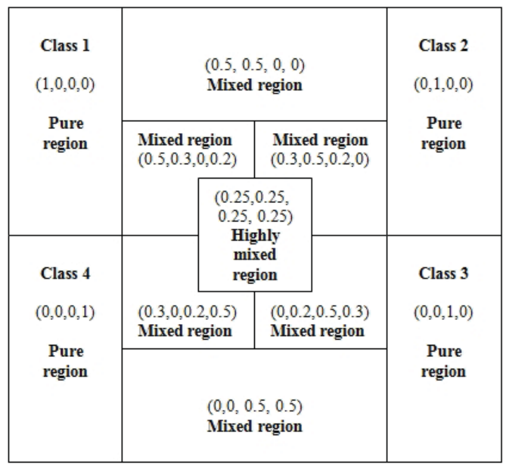


Figure 1: Proportion values of each class for synthetic data generation

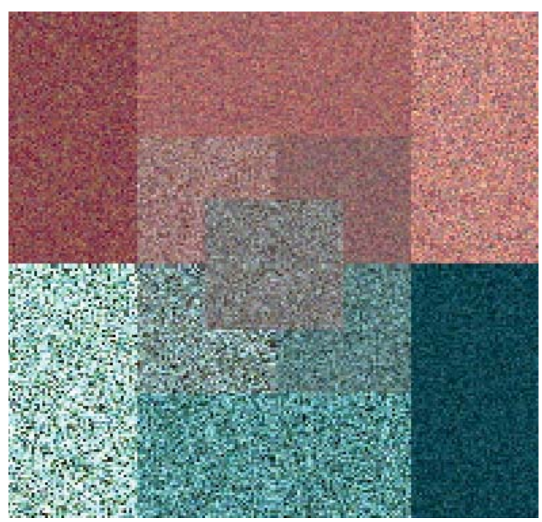
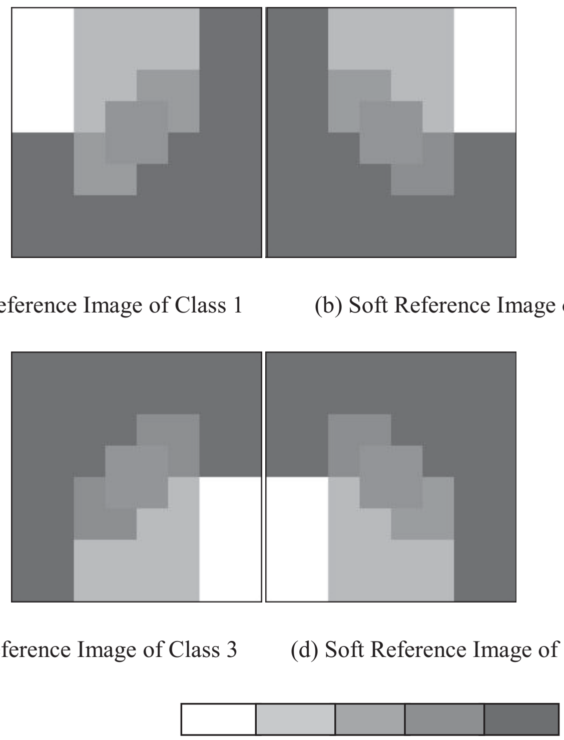


Figure 2: False colour composite image used for classification
(Red: band 5, Green: band 3, Blue: band 2)

On the basis of univariate statistical properties of individual class in each spectral band, it was found that the generated synthetic images are consistent with the actual remote sensing data acquired in a region, where classes are generally mixed and overlapping. The data consists of about 45% of mixed pixels, which contribute to the aspects of classification quality to a large extent. From the transformed divergence analysis conducted on all the six bands in synthetic dataset, a TD value of 2000 was obtained for the combination of band 2 (0.52-0.6 µm), band 3 (0.63-0.69 µm) and band 5 (1.55-1.75 µm). Therefore, these three bands were used as input into the classifier selected for deriving class membership values. Figure 2 shows the false colour composite (FCC) image considered for the study.



(a) Soft Reference Image of Class 1 (b) Soft Reference Image of Class 2

(c) Soft Reference Image of Class 3 (d) Soft Reference Image of Class 4

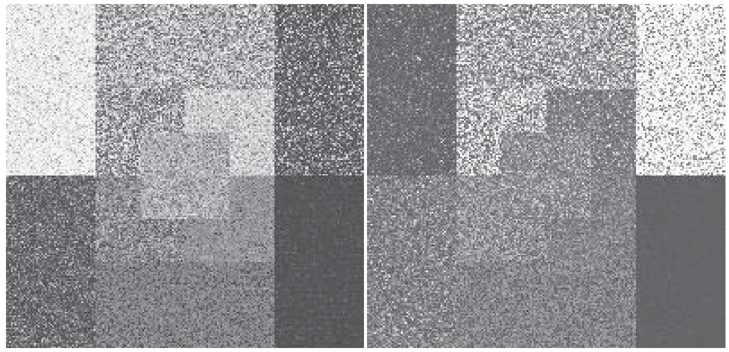
Pixel value 1 0.5 0.3 0.25 0

Figure 3: Soft reference images of each class

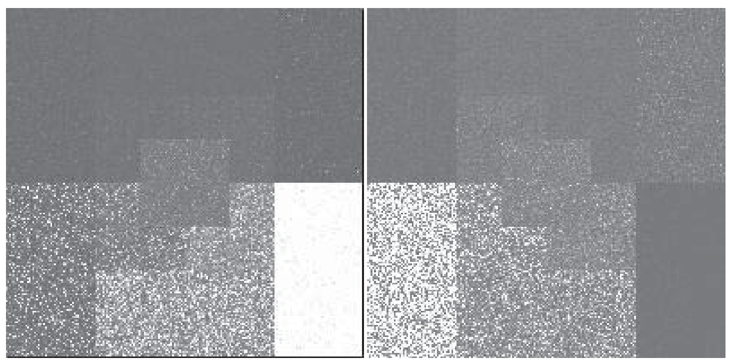
The actual class proportion values (Figure 1) used to generate synthetic images have been further used to create soft reference images for each class. The class proportion values for each pixel in all the regions have been named as soft reference data and are represented as fraction images for each class (Figure 3).

Fuzzy c-means classifier described by Bezdek et al. (1984) was adopted to produce a soft classification of the study image. This algorithm has proven especially popular (Legleiter and Goodchild, 2005; Bastin, 1997; Wu and Yang 2002; Yang et al., 2003) and has been used to produce land cover maps from remotely sensed data (Zhang and Stuart, 2001). In most situations the fuzzy c-means classifier may be advantageous (Shalan et al., 2003) as it is not dependent on the data distributional assumptions. The fuzzy c-means classifier is based on an iterative clustering algorithm which partitions pixels in the image into class proportions. Although, it is an unsupervised classifier (Bezdek et al., 1984), it may be used in supervised mode (Foody, 2000). The formulation of fuzzy c-means classifier contains a weighting factor m, which describes the degree of fuzziness to be introduced in the classification. The value of m varies from 1 (no fuzziness or hard classification) to ∞ (complete fuzziness). A value in the range of 1.5 to 3 may generally be adopted (Shalan et al., 2003). Through several experiments, a value of m=2.0 was found to be suitable for the classification of this dataset. The number of training samples were kept as 200 for each of the classes and were selected in the pure regions.

The results of the classification were four fraction images (Figure 4) corresponding to four classes considered in the experiment. In Figure 4, bright pixels indicate higher class membership values. And, darker pixels correspond to lower values of class membership. On visual comparison of these fraction images with the corresponding fraction images from soft reference data, deviation of the derived membership values from the reference can be observed. For example, this deviation is more in classes 1, 2 and 4 than in the class 3, which implies that these three classes (1, 2 and 4) may be more uncertain than the other (class 3).



(a) Fraction image for class 1 (b) Fraction image for class 2



(c) Fraction image for class 3 (d) Fraction image for class 4

Figure 4: Fraction images derived from fuzzy classification

4. Result and discussion

Since the experimental data set had varied spectral characteristics and sufficient proportion of mixed pixels with desired 'within class variability' and 'boundary effect', the result of the classification was satisfactory, though not excellent. When, the accuracy of classification was estimated using fuzzy error matrix approach, the overall accuracy of classification was found to be 68.4%.

Table 1: Values of measures of classification quality derived from fuzzy membership values of classification and reference

Class	Measure of Similarity	Fuzzy certainty measure	Goodness of fit
1	68.04 %	80.40 %	80.20 %
2	66.36 %	83.10 %	82.90 %
3	77.31 %	89.45 %	89.20 %
4	61.79 %	84.50 %	84.30 %

To quantify classification quality using the measure of similarity (Eq. 4) and fuzzy certainty measure (Eq. 5), the derived fuzzy membership values in each of the classes and the corresponding reference data have been used as input to computational models. Values of the quality indicators used in the present study have been computed class wise for better understanding of the performance of the individual measure considered. The results (in %) for individual classes are presented in Table 1.

From table 1, it is observed that the quality of classification for class 3 appears to be better than other three classes. Both the measures are indicating the same with the higher values (77.31% and 89.45%) when compared to other values. However, in the case of class 4, the value of the measure of similarity is the lowest, while the fuzzy certainty measure has produced the lowest value for class 1. The values estimated by fuzzy certainty measure for all the four classes appears to be high indicating good match between the classification and the reference. This may not be the true case as the overall accuracy of this classification as estimated from fuzzy error matrix is only 68.4%. And, the producer's accuracy values for the classes 1, 2, 3 and 4 were found to be 36.7%, 36.7%, 43.4%, and 43.6% respectively. Further, the fraction images from classification and reference were defuzzified and kappa index of agreement values for all the four classes were determined as 0.4372, 0.4197, 0.8428 and 0.5738 respectively. Therefore, it seems that the values estimated by the measure of similarity are reasonable, when compared to those from fuzzy certainty measure.

Further, In order to assess the appropriateness of the two measures considered a Goodness of Fit measure (G) widely used in statistical regressions has been computed. This Goodness of Fit (G) may be defined as scaled absolute difference and is given by,

$$\text{Goodness of fit (G)} = 1 - \frac{|\mu_{i,REF} - \mu_{i,CLASS}|}{MAX\{|\mu_{i,REF} - \mu_{i,CLASS}|\}} \quad (6)$$

The numerator term in Eq. (6) produces a positive value and the denominator in Eq.(6) is a scaling factor such that the *G* values for a pixel lie between zero and one.

This measure has been used to compare fraction images from fuzzy classification and soft reference data on pixel by pixel basis. The minimum value for *G* is zero, which indicates complete mismatch between two data sets, a *G* value of 1 indicates 100% matching between the two. Values of G for all the four classes are also shown in table 1. By observing the values of G and the values of fuzzy certainty measure in table 1, it is evident that the fuzzy certainty measure is estimating values almost identical to those from goodness of fit measure.

Fuzzy error matrix based measures use "MIN' operator to obtain the cardinality of fuzzy set intersection which provide global values. It is to be noted here that, it is not possible to derive producer's or user's accuracy from fuzzy similarity measures, the reason being that both of them are not symmetrical (Jager and Benz, 2000). Fuzzy similarity measure used in this study also uses the similar operator and hence able to produce reasonable values as in the case of fuzzy error matrix based measures. While fuzzy certainty measure is typically behaving like a statistical measure which summarizes the discrepancy between the classification and the reference values and may be considered as another facet of goodness of fit. One must be careful in using such goodness of fit measures and two issues must be considered. First, there are inherent errors in the classification results and in the reference data also. Secondly, the effect of lower membership values in the classification in predicting the overall goodness of fit which may be very high in such cases.

To provide information on quality of classification at pixel level, the measures used in this study require information from the reference data at the same scale. Availability of error free soft reference data or creation of such data is another problem. Very often the soft reference data may be from another classification based on another data set. Thus, the error-prone or dubious reference data may also have an impact on the assessment of quality using fuzzy set based accuracy or similarity measures. Often, accuracy metrics alone would not be sufficient to infer about the quality. Therefore, methods which complement the conventional accuracy measures are required to represent the quality of classification at pixel scale. Few studies have used confidence as a measure of classification quality (McIver and Friedl, 2001; Liu et al., 2004; Ganesh Prasad and Arora, 2014). In the absence of fuzzy reference data, the quality of a fuzzy classification can be estimated by using a simple measure of confidence proposed by Ganesh Prasad and Arora (2014). In recent years, impetus has been placed on assessment of uncertainty in spatial data particularly in remote sensing derived land use-land cover maps and seems to be a rapidly growing research area. Uncertainty as a quality assessment tool has become a key subject in remote sensing studies and has attracted attention of many researchers. Presenting uncertainty information in addition to global values of classification accuracy may provide enhanced quality information to the users in assessing the fitness for use of maps derived from image classification.

5. Summary and conclusions

The use of soft classifications to produce accurate and reliable land cover maps is gaining momentum and therefore, there is a need to adopt a suitable measure which can indicate the overall quality of a fuzzy classification at the pixel level. Many measures for evaluating the quality of fuzzy classification based on distances, fuzzy error matrix and fuzzy similarity measures have been proposed. However, none of these measures have been accepted as a universal measure for evaluating the quality of a fuzzy classification with fuzzy reference data. Many users of thematic maps derived from remotely sensed data may be benefited by providing quality information at the pixel level. The present study aimed at applying two measures viz., fuzzy similarity measure and fuzzy certainty measure for assessing the quality of a fuzzy classification. Fuzzy *c* means classification was applied to a synthetic data set to derive fuzzy membership values. The derived fuzzy membership values and the corresponding fuzzy reference data were used to compute the values of fuzzy similarity measure and fuzzy certainty measure for each of the classes. The results indicated that the two measures estimate the values differently. However, fuzzy certainty measure produced values identical to those from a simple measure of goodness of fit used in statistical regression.

The users of thematic maps derived from fuzzy classification of remotely sensed data may get benefited with the information regarding the quality of classification. However, the availability of error free fuzzy reference data plays an important role in evaluating the quality of soft classification in terms of accuracy and similarity. Therefore, it would be beneficial to users, if the quality of soft classification is reported not only in terms of accuracy, but with supplementary information provided by appropriate classification uncertainty and confidence measures.

References

Bastin, L. (1997). Comparison of fuzzy c-means classification, linear mixture modelling and MLC probabilities as tools for unmixing coarse pixels.

International Journal of Remote Sensing, 18(17), 3629-3648.

Bezdek, J.C., R. Ehrlich and W. Full (1984). FCM: The fuzzy c-means clustering algorithm. Computers and Geosciences, 10, 191-203.

Binaghi, E., P.A. Brivio, P. Ghessi and A. Rampini (1999). A fuzzy set based accuracy assessment of soft classification. Pattern Recognition Letters, 20, 935-948.

Brown, M., H.G. Lewis and S.R. Gunn (2000) Linear spectral mixture models and support vector machines remote sensing. IEEE Transactions on Geosciences and Remote Sensing, 38, 2346-2360.

Foody, G.M. (1995) Cross- entropy for the evaluation of the accuracy of a fuzzy land cover classification with fuzzy ground data. ISPRS Journal of Photogrammetry and Remote Sensing, 50, 2-12.

Foody G.M. (1996) Relating the land cover composition of mixed pixels to artificial neural network classification output. Photogrammetric Engineering and Remote Sensing, 62, 491-499

Foody, G.M. and M.K. Arora (1996) Incorporating mixed pixel in the training, allocation and testing stages of supervised classification. Pattern Recognition Letters, 17, 1389-1398.

Foody, G.M. (2000) Estimation of sub-pixel land cover composition in the presence of untrained classes. Computers and Geosciences, 26, 469-478.

Ganesh Prasad, M.S. and M.K.Arora, (2014). A simple measure of confidence for fuzzy land cover classification from remote sensing data. International Journal of Remote Sensing, 35(24), pp. 8122-8137.

Ganesh Prasad, M.S. (2015). Multi-spectral synthetic image generation using linear mixture model. Journal of Computing Technologies, 4(6).

Gomez, D., G. Biging and J. Montero (2008). Accuracy statistics for judging soft classification. International Journal of Remote Sensing, 29(3), 693-709.

Gopal, S. and C.E. Woodcock (1994). Theory and methods for accuracy assessment of thematic maps using fuzzy sets. Photogrammetric Engineering and Remote Sensing. 60, 181-188.

Jäger, G. and U. Benz (2000). Measures of classification accuracy based on fuzzy similarity. IEEE Transactions on Geoscience and Remote Sensing, 38(3), 1462-1467.

Laba, M, S. Gregory, J. Braden, D. Ogurcak, E. Hill, E. Fegraus, J. Fiore and S.D. Degloria (2002). Conventional and fuzzy accuracy assessment of the New York Gap Analysis Project land cover map. Remote Sensing of Environment 82, 443-456.

Latifovic, R. and I. Olthof (2004). Accuracy assessment using sub-pixel fractional error matrices of global land cover products derived from satellite data. Remote Sensing of Environment, 90, 153-165.

Legleiter, C.J. and M.F. Goodchild (2005). Alternative representations of in-stream habitat: classification using remote sensing, hydraulic modeling, and fuzzy logic. International Journal of Geographical Information Science, 19(1), pp.29-50.

Liu, W., S. Gopal and C.E. Woodcock (2004). Uncertainty and confidence in land cover classification using a hybrid classifier approach. Photogrammetric Engineering and Remote Sensing, 70, 963-972.

Maselli, F., C. Conese and L. Petkov (1994). Use of probability entropy for the estimation and graphical representation of the accuracy of maximum likelihood classification. ISPRS Journal of Photogrammetry and Remote Sensing, 49, pp.13-20.

Maselli, F., A. Rodolf and C. Conese (1996). Fuzzy classification of spatially degraded thematic mapper data for the estimation of sub-pixel components. International Journal of Remote Sensing, 17, 537-551.

McIver, D.K. and M.A. Friedl (2001). Estimating pixel scale land cover classification confidence using nonparametric machine learning methods. IEEE Transactions on Geoscience and Remote Sensing, 39, 1959-1968.

Okeke, F. and A. Karnieli (2006). Methods for fuzzy classification and accuracy assessment of historical aerial photographs for vegetation change analyses. Part I: Algorithm development. International Journal of Remote Sensing, 27, 153-176.

Pontius Jr R.G. and M.L. Cheuk (2006). A generalized cross-tabulation matrix to compare soft-classified maps at multiple resolutions. International Journal of Geographical Information Science, 20 (1), 1-30.

Ricotta, C. and G.C. Avena (2002). Evaluating the degree of fuzziness of thematic maps with a generalized entropy function: A methodological outlook. International Journal of Remote sensing, 23(20), 4519-4523

Ricotta, C. (2004). Evaluating the classification accuracy of fuzzy thematic maps with a simple parametric measure. International Journal of Remote Sensing, 25, 2169-2176.

Schiewe, J. and M. Gähler (2008). Modelling uncertainty in high resolution remotely sensed scenes using a fuzzy logic approach. In Object-Based Image Analysis (pp. 755-768). Springer Berlin Heidelberg.

Settle, J.J. and N.A. Drake (1993). Linear mixing and the estimation of ground cover proportions. International Journal of Remote Sensing, 14, 1159-1177.

Shalan, M.A., M.K. Arora and S.K. Ghosh (2003). An evaluation of fuzzy classifications from IRS 1C LISS III imagery : A case study. International Journal of Remote Sensing. doi:10.1080/0143116031000094791

Silvan-Cardenas, J.L. and L. Wang (2008). Sub-pixel confusion-uncertainty matrix for assessing soft classification. Remote Sensing of Environment, 112, 1081-1095.

Stehman, S.V., M.K. Arora, K. Teerasit and P.K. Varshney (2007). Estimation of fuzzy error matrix accuracy measures under stratified random sampling. Photogrammetric Engineering and Remote Sensing, 73(2), 165-173.

Tran, L.T., J. Wickham, S.T. Jarnagin and C.G. Knight (2005). Mapping spatial thematic accuracy with fuzzy sets. Photogrammetric Engineering and Remote Sensing, 71(1), 29-36.

Varshney, P.K. and M.K. Arora (2004). Advanced image processing techniques for remotely sensed hyper spectral data. Springer Verlag ,Heidelberg.

Woodcock, C.E. and S. Gopal (2000). Fuzzy set theory and thematic maps: accuracy assessment and area estimation. International Journal of Geographical Information Science, 14, 153-172.

Wu, K.L. and M.S. Yang (2002). Alternative c-means clustering algorithms. Pattern Recognition. 35, 2267-2278.

Yang, M.S., P.Y. Hwang and D.H. Chen (2003). Fuzzy clustering algorithms for mixed feature variables. Fuzzy Sets and Systems, 141, 301-317.

Zhang, J. and N. Stuart (2001). Fuzzy methods for categorical mapping with image based land cover data. International Journal of Geographic Information Science, 15(2), 175-195.