Horizontal coordinate transformation using artificial neural network technology: A case study of Ghana geodetic reference network

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Abstract: Transformation of coordinates between different geodetic datums has been a common practice within the geospatial profession. Relating different geodetic datums mostly involves the use of conformal transformation techniques which could produce results that are not very often satisfactory for certain geodetic, surveying and mapping purposes. This has been attributed to the inability of the conformal models to absorb more of the heterogeneous and local character of deformations existing within the local geodetic networks. In light of this, most researchers have resorted to Artificial Neural Network (ANN) as a plausible alternative technology for coordinate transformation. Although the ANN technique has been applied successfully, the method is yet to be adopted and tested within the Ghana geodetic reference network. In view of this, the present study applied the Radial Basis Function Neural Network (RBFNN) to transform plane coordinates between the two classical geodetic networks namely, Accra datum and Legion datum which is used in Ghana for its surveying and mapping activities. The RBFNN results obtained were compared with the four-parameter and six-parameter transformation models. It was noticed that the RBFNN could produce more reliable and accurate results and thus is more applicable in modelling distortions existing in the local geodetic networks more effectively than the traditional techniques (six-parameter and four-parameter).

Keywords: Radial basis function neural network, Four-parameter model, Six-parameter model, Artificial neural network

1. Introduction

Coordinate transformation is a common problem faced by the geospatial practitioners. Its significance is to enable transformation of coordinate from a separate survey into a common system thereby integrating data from different sources on to the same reference surface. One of the most common examples of these reference surfaces is the geometric reference ellipsoid. This ellipsoid serves as the horizontal datum for two or three-dimensional positions obtained via Global Positioning System (GPS) and control survey methods such as triangulation, trilateration, resection (Schofield, 2001) and many others for both developed and developing countries such as Ghana.

Currently, there are two reference ellipsoids that are being utilized by the geospatial professionals in Ghana. These are the War Office 1926 ellipsoid built on the Accra datum 1920 and the Clark 1880 (modified) ellipsoid built on the Legion datum1977. Issues have been raised in Ghana about the implementation of these two datums for survey and mapping purposes. For instance, it was pointed out by Ayer and Fosu (2008) that the discrepancies between War Office and Clark 1880 (modified) ellipsoid is about 26 m which does not meet the requirements of ± 0.9114 m stipulated by the Survey and Mapping Division of Lands Commission in Ghana. The large error value could be attributed to the different geometric parameters of the two local reference ellipsoids. This has therefore necessitated the need to have an integrated approach to solve the issue of discrepancies between the War Office1926 and Clark 1880 ellipsoid in Ghana. One of the proposed ways is to determine coordinate transformation parameters between the two local ellipsoids namely, War Office1926 and Clark 1880 (modified). However, conformal coordinate transformation models which have mostly been frequently used could not produce satisfactory results (Grigic et al., 2015). Most researchers have attributed this limitation to the heterogeneous nature of local geodetic networks contributed by the methods used for its establishment (Tierra et al., 2008).

In order to correct for such defects in the conformal models, artificial intelligence method such as Artificial Neural Network (ANN) has been resorted to by several researchers in recent times. For instance, the Radial Basis Function Neural Network (RBFNN) has been widely used and compared to other transformation models such as the 3-parameter, standard Molodensky, Abridged Molodensky, Bursa-Wolf, 2D conformal model, 2D Affine, Molodensky-Badekas and many others (Tierra et al., 2008; Tierra et al., 2009; Gullu et al., 2011; Gullu 2010). Likewise, the Backpropagation Neural Network (BPNN) has also been tested in coordinate transformation (Tierra and Romero, 2014; Mihalache, 2012; Turgut, 2010; Yilmaz and Gullu, 2011; Zaletnyik, 2004; Lin and Wang, 2006). In addition, an investigation into a genetic based method for directly transforming 2D coordinates has also been studied (Chih-Hung et al., 2008). Furthermore, a preliminary study on the concept of the recurrent cascade neural networks and the Neuro-Fuzzy Neural Network (NFNN) based on the Takagi-Sugeno-Kang system for coordinate transformation has also been explored (Gil and Mrowczynska, 2012). The conclusions drawn from the various studies indicate that
the artificial intelligence methods are dominant over the traditional transformation techniques and thus could serve as a better alternative approach for coordinate transformation.

Whereas the artificial intelligence methods have been gainfully applied, literature covered pertaining to this study revealed that, in Ghana, only the three-dimension (3D) similarity transformation models have been utilized to transform from global (WGS84) datum to local datums (Accra and Leigon). The most commonly used 3D coordinate transformation methods in Ghana are the conformal similarity models of Bursa-wolf, Molodensky-Badekas, Abridged Molodensky, Veis model, 3D affine model and 3D projective model (Ayer, 2008; Ayer and Fosu, 2008; Dzidefo, 2011; Kotzev, 2013; Ziggah et al., 2013 a and b). Thus, no research work has been done to establish transformation parameters between War Office 1926 and Clark 1880 ellipsoidal datums used in Ghana. This is because Ayer and Fosu (2008) only showed the differences between the local ellipsoids with no parameters determined to unify the two geodetic datums. This implies that determining parameters to assimilate War Office 1926 and Clark 1880 ellipsoids data has not yet been fully investigated. In addition, the recent artificial neural network technology is yet to be adopted.

Therefore, this study focuses on Ghana as a case of application for its national geodetic network and proposes artificial neural network as a viable approach for integrating data from War Office 1926 to Clark 1880 (modified) ellipsoid and vice versa.

2. Study area and data source

The study area is Ghana located in West Africa. It is bordered on the North by Burkina Faso, Ivory Coast to the West, Togo to the East and Gulf of Guinea to the South. Ghana uses two horizontal datums known as the Accra and Leigon datum for its geodetic activities. The reference surface of the Accra datum is the War Office 1926 ellipsoid suggested by the British War Office, with semi-major axis \( a = 6378299.99899832 \) m, semi minor axis \( b = 6356751.66824042 \) m, flattening \( f = 1/296 \) (Ayer, 2008; Ayer and Fosu, 2008). On the other hand, Leigon datum has the Clark 1880 (modified) ellipsoid as its reference surface, with semi-major axis \( a = 6378249.145 \) m, semi minor axis \( b = 6356514.870 \) m, flattening \( f = 1/293.465006079115 \). The plane coordinates are estimated on the Transverse Mercator projection with central meridian at longitude 1° W (Mugnier, 2000; Poku-Gyamfi and Hein, 2006). It is important to note that, the coordinate system used to indicate positions of features on all survey maps in Ghana is the projected grid coordinates of Easting and Northing derived from the Transverse Mercator projection.

In this study, secondary data of 27 common plane coordinate points in Easting and Northing in both Accra datum and Leigon datum were obtained from the Survey and Mapping Division of Lands Commission in Ghana. These acquired co-located points are the historical triangulation positions that were used as the realisation of the War Office 1926 and Clark 1880 (modified) reference systems. Records from the Ghana Survey and Mapping Division indicate that these historical triangulation points used in this study covers the mountainous southern regions of Ghana and thus were established on hilly terrain (Kotzev, 2013). These 27 historical triangulation points provided covers the area of the on-going GNSS national geodetic reference network establishment under the Land Administration Project (LAP) sponsored by the World Bank (Kotzev, 2013). One of the objectives for the LAP is to establish GNSS geodetic reference network to enhance the use of Global Navigation Satellite System (GNSS) for land related positioning undertakings such as geotechnical investigations, traffic and transportation, meteorology, survey and mapping, timing, engineering and many others in Ghana (Wonnacot, 2007; Poku-Gyamfi and Schueler, 2008). Furthermore, the LAP tend to achieve the proposed unification of all national reference frames for all African countries under the African Reference Frame into a single continental reference system based on the International Terrestrial Reference System (ITRS). The southern section where these historical triangulation points are situated has been termed the golden triangle. Figure 1 shows the data distribution of the historical points in the golden triangle.

![Figure 1: Study area showing geographic data distribution](image)
3. Methods

3.1 Four-parameter similarity model

The four-parameter similarity model is composed of two translations of the coordinate origin, one scale factor and one rotation parameter. This model was applied in this study to determine parameters suitable to transform coordinates from War Office 1926 ellipsoid to Clark 1880 (modified) ellipsoid and vice versa. It is stated by Ghilani (2010) that the four parameter model is mostly suitable when converting separate surveys into a common reference coordinate system. Moreover, Deakin (2006) also emphasised that the model preserves shape and angles after transformation and hence is a useful tool for mapping activities. The four-parameter similarity model relates two rectangular coordinate systems expressed in Equation 1 (Ghilani, 2010) as

\[ \begin{align*}
ax - by + c &= X \\
bx + ay + d &= Y
\end{align*} \]

(1)

Applying the least squares method, Eq. 1 could be represented in matrix form (Eq. 2) as

\[ V + BX = L \]

(2)

where \( V \) is the residual, \( B \) is the designed matrix, \( L \) is the observation vector matrix and \( X \) is the vector of the unknown parameters to be determined.

Hence, expressing Eq. 1 in the form of Eq. 2 gives Eq. 3 expressed as

\[ \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a & b & V_x \\ c & d & V_y \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} \]

(3)

where

\[ B = \begin{bmatrix} x & -y & 1 \\ y & x & 0 \end{bmatrix} \]

is the design matrix;

\[ X = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \]

is the vector of the unknown transformation parameters to be determined;

\[ V = \begin{bmatrix} V_x \\ V_y \end{bmatrix} \]

is the residual; and \( L = \begin{bmatrix} X \\ Y \end{bmatrix} \) is the observation vector respectively.

3.2 Six-parameter transformation model

The six-parameter transformation model consists of two translations of the origin, two different scale factors; one in the \( x \) direction and the other in the \( y \) direction. Also there is a rotation about the origin, plus a small non-orthogonality correction between the \( x \) and \( y \) axes (Ghilani, 2010). This model was also used in this study to calculate parameters suitable to transform coordinates from War Office 1926 ellipsoid to Clark 1880 (modified) ellipsoid and vice versa.

The observation equations for the six-parameter transformation are given in Eq. 4 (Ghilani, 2010)

\[ \begin{align*}
ax + by + c &= X \\
dx + ey + f &= Y
\end{align*} \]

(4)

Equation 4 when expressed in the matrix form (Eq. 2) is given by Eq. 5 as

\[ \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = \begin{bmatrix} X \\ Y \end{bmatrix} + \begin{bmatrix} V_x \\ V_y \end{bmatrix} \]

(5)

where:

\[ B = \begin{bmatrix} x & y & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x & y & 1 \end{bmatrix}, \]

\[ L = \begin{bmatrix} X \\ Y \end{bmatrix}, \quad V = \begin{bmatrix} V_x \\ V_y \end{bmatrix} \]

and \( X = \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} \) respectively.

3.3 Radial Basis Function Neural Network (RBFNN)

The RBFNN is a three layered topology namely, input, hidden and output layer that are completely linked together in a feed forward manner. The input layer receives the input data information while the output layer produces the final computed results. The layer that does not have direct access to the external world is known as the hidden layers. Figure 2 shows the RBFNN architecture with inputs \( (X_i) \), radial basis function \( (\phi_n) \), weight \( (W_N) \) and output \( (y) \) respectively.
In the hidden layer chamber, the input layer data is received by means of unweighted connections. The data is then transformed by means of a non-linear activation function with each neuron estimating a Euclidean norm that shows the distance between the input to the network and the position of the neuron called the centre. This is then inserted into a radial basis activation function which calculates and outputs the activation of the neuron (Deyfrus, 2005). The present study applied the Gaussian activation function (Gurney, 2005) expressed in Eq. 6 as

$$a_j = \varphi_j(X) = \exp \left[ - \frac{\|X - \mu_j\|^2}{2\sigma_j^2} \right]$$

where $X$ is the input vector, $\mu_j$ is the centre of the Gaussian function and $\sigma_j$ is the spread parameter of the Gaussian function bell and $\|\|$ is the Euclidean norm.

The output layer contains the linear function and uses the weighted sum of the hidden layer as propagation function expressed in Eq. 7 (Tierra et al., 2008) as

$$Y_k = \sum_{j=1}^{p} W_{jk} a_j + b_0.$$  

Here, each $W_{jk}$ is the output weight that matches to the association between a hidden node and an output node while $b_0$ is the bias that has a unit weight and $p$ denotes the number of hidden neurons. Equation 7 could be achieved by the RBFNN through learning. Hence, in this study, the supervised learning paradigm was used to establish the input-output mapping functions. Thus, in a supervised application a set of data samples called training for which the corresponding desired outputs are known is fed into the network. It is important to note that the training can be characterized as a non-linear optimization problem where the network parameters are to be solved (Barsi, 2001) such that the estimated outputs from the RBFNN will be in good agreement with the target data.

The error $e_k$ in the output of a neuron $k$ is defined as the deviation of the desired value $d_k$ from the computed value $Y_k$ in the first step (Haykin, 1999). This is expressed (Eq. 8) by

$$e_k = d_k - Y_k.$$  

The training process continues until the network error reaches an acceptable value.

### 4. Accuracy assessment

In order to compare the results attained by the RBFNN model with the six-parameter and four-parameter models, the residuals computed between the desired plane coordinates and the transformed plane coordinates based on the test data set was used. The performance indicators utilized include the Mean Squared Error (MSE), Mean Bias Error (MBE), Mean Absolute Error (MAE), Horizontal Position Error (HE), Mean Horizontal Position Error (MHE) and Standard Deviation (SD). The mathematical expression for the various performance indices are given by Eq. 9 to 14 respectively.

$$MSE = \frac{1}{n} \sum_{i=1}^{n} (O_i - P_i)^2;$$

$$MBE = \frac{1}{n} \sum_{i=1}^{n} (O_i - P_i);$$

$$MAE = \frac{1}{n} \sum_{i=1}^{n} |O_i - P_i|;$$

$$HE = \sqrt{(E_2 - E_1)^2 + (N_2 - N_1)^2};$$

$$MHE = \frac{1}{n} \sum_{i=1}^{n} HE_i;$$

$$SD = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (e_i - \bar{e})^2};$$

where $n$ is the total number of test examples presented to the learning algorithm; $O$ and $P$ are the measured and predicted plane coordinates from the various procedures; $e$ represents the deviations between the measured and predicted plane coordinates; and $\bar{e}$ is the mean of the deviations.

### 5. Test results

The 27 common points (Figure 3) coordinate in Accra datum and Leigon datum for the Ghana national geodetic reference network was divided into reference data set and testing data set. Twenty points were selected as the reference points $(P_1, P_2, \ldots, P_{20})$ while the testing data set consisted of 7 points $(T_1, T_2, \ldots, T_7)$. In this study, the chosen reference coordinates were used to determine the transformation parameters based on the four and six-parameter models as well as the RBFNN training process in the ANN approach. The test data served as an independent check on the performance of the afore mentioned techniques with the test results presented in the following sections.
The MSE was then used as the criterion to determine the optimum RBFNN structure during the training process. After several trials, the optimum structure of the RBFNN for transforming coordinates from Leigon datum to Accra datum was [2-20-2]. Thus, two inputs (E\textsubscript{clark}, N\textsubscript{clark}) with twenty hidden neurons and two outputs (E\textsubscript{war}, N\textsubscript{war}).

### Table 1: Six parameter model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.99999</td>
<td>3.20E-06</td>
</tr>
<tr>
<td>B</td>
<td>-1.13E-05</td>
<td>4.23E-06</td>
</tr>
<tr>
<td>C</td>
<td>2.22345</td>
<td>1.417531</td>
</tr>
<tr>
<td>D</td>
<td>1.75E-05</td>
<td>3.20E-06</td>
</tr>
<tr>
<td>E</td>
<td>1.00001</td>
<td>4.23E-06</td>
</tr>
<tr>
<td>F</td>
<td>0.50298</td>
<td>1.417531</td>
</tr>
</tbody>
</table>

### Table 2: Four parameter model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>1.000001</td>
<td>2.06E-06</td>
</tr>
<tr>
<td>R</td>
<td>1.27E-05</td>
<td>2.06E-06</td>
</tr>
<tr>
<td>\Delta X</td>
<td>2.080065</td>
<td>0.692668</td>
</tr>
<tr>
<td>\Delta Y</td>
<td>2.98E+00</td>
<td>0.693</td>
</tr>
</tbody>
</table>

Table 3 shows how much the estimated plane coordinates from the existing plane coordinates. The HE, mean error and SD values for each test coordinate are also presented. As indicated earlier at the study area and data source section, these historical triangulation points cover the mountainous southern section of Ghana and thus were established on hilly terrain. Hence, the analyses presented henceforth should be viewed as a way of assessing the applicability of the RBFNN, four-parameter and six-parameter models for transforming 2D national coordinates between the Accra and Leigon datums based on hilly terrain data.

### Table 3: Deviation of transformed test coordinates from observed coordinates

<table>
<thead>
<tr>
<th>Test Point</th>
<th>SIX PARAMETER</th>
<th>FOUR PARAMETER</th>
<th>RBFNN</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>\Delta E(m)</td>
<td>\Delta N(m)</td>
<td>HE (m)</td>
</tr>
<tr>
<td>T1</td>
<td>-0.14795</td>
<td>1.39339</td>
<td>1.40122</td>
</tr>
<tr>
<td>T2</td>
<td>0.638301</td>
<td>0.62816</td>
<td>0.89555</td>
</tr>
<tr>
<td>T3</td>
<td>0.464141</td>
<td>-1.33447</td>
<td>1.41289</td>
</tr>
<tr>
<td>T4</td>
<td>0.32133</td>
<td>0.07592</td>
<td>0.33018</td>
</tr>
<tr>
<td>T5</td>
<td>0.601158</td>
<td>0.48968</td>
<td>0.77536</td>
</tr>
<tr>
<td>T6</td>
<td>-1.51539</td>
<td>0.33729</td>
<td>1.55247</td>
</tr>
<tr>
<td>T7</td>
<td>-0.22456</td>
<td>0.81286</td>
<td>0.84331</td>
</tr>
<tr>
<td>Mean Error</td>
<td>0.01958</td>
<td>0.34326</td>
<td>1.03014</td>
</tr>
<tr>
<td>SD</td>
<td>0.75868</td>
<td>0.84776</td>
<td>0.44063</td>
</tr>
</tbody>
</table>
The deviations (ΔE, ΔN) (Table 3) known as errors indicate the variation between the transformed plane coordinates and the measured plane coordinates relative to the ideal zero error value. Analysis of Table 3 indicates that the RBFNN predicted outcomes in both Eastings and Northings are in good agreement with the measured plane coordinates compared with the six and four-parameter models. In order to ascertain the practicality of the transformed plane coordinates from both methods, their horizontal positional accuracies were assessed. This was done by estimating the amount of HE from the transformed plane coordinates.

Figure 4 shows the HE for the results obtained by the six-parameter model, four-parameter model and RBFNN. Based on visual observation of Figure 4 and Table 3, it can be seen that, the six-parameter, four-parameter and RBFNN methods predicts the horizontal errors with a minimum uncertainty in the order of approximately 0.33 m, 0.39 m and 0.19 m respectively. On the other hand, the maximum horizontal displacement of about 1.55 m, 1.32 m and 0.8 m were known for the six-parameter, four-parameter and RBFNN.

The reason for improvement in the horizontal positional accuracies attained by the RBFNN might possibly be that the distortions existing in both datums established by conventional surveying techniques have been modelled and more absorbed by the RBFNN model compared with the four-parameter and six-parameter models. Hence, the inference to be made here is that the influence of the local geodetic network distortions on the coordinate transformation results has been minimized by the RBFNN model thereby improving the transformation accuracy.

![Figure 4: Horizontal displacement of test coordinates](Image)

Table 4: Difference in the horizontal positional accuracy between the traditional techniques and RBFNN

<table>
<thead>
<tr>
<th>Test Point</th>
<th>Six parameter (m)</th>
<th>Four parameter (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>1.07196</td>
<td>0.57964</td>
</tr>
<tr>
<td>T2</td>
<td>0.56577</td>
<td>0.87931</td>
</tr>
<tr>
<td>T3</td>
<td>1.22353</td>
<td>0.88076</td>
</tr>
<tr>
<td>T4</td>
<td>-0.13955</td>
<td>-0.08184</td>
</tr>
<tr>
<td>T5</td>
<td>0.41726</td>
<td>0.3726</td>
</tr>
<tr>
<td>T6</td>
<td>0.73102</td>
<td>0.49732</td>
</tr>
<tr>
<td>T7</td>
<td>0.0722</td>
<td>0.50561</td>
</tr>
</tbody>
</table>

With reference to Table 4, it can be seen that the four and six-parameter HE values deviated from the RBFNN results on a higher level. This implies that the traditional techniques could not absorb more of the distortions within the two local geodetic networks as compared to the RBFNN. However, a close inspection of Table 4 and Figure 3 shows that only the test point T4 HE value for the six-parameter and four-parameter model was approximately 0.14 m and 0.08 m better than the RBFNN model transformed value. This could be due to the inability of the RBFNN model to absorb more of the distortion in the T4 contributing to the calculated HE value of T4 being a little higher. This is because HE is highly dependent on the horizontal position coordinates for its estimation. This further confirms the assertion made by Featherstone (1997) that mathematical transformation models cannot completely account for the distortions in local geodetic network and thus geocentric datum adoption should be the preferred choice. Nonetheless, overall, in the present study the RBFNN gave more reasonable and applicable transformed values than the six and four-parameter models. Hence, the RBFNN results could be used as an interim solution until the adoption of a geocentric datum in Ghana.

In Table 5, a summary of the total error attained when the three methods were applied for transforming coordinates from Leigon to Accra datum is presented. It can be observed from Table 5 that there is a significant improvement in the results for the RBFNN in all the Performance Criteria Indices (PCI) utilized as compared with the other two techniques. The inference made in line with the maximum and minimum values (Table 5) is that the RBFNN model predicted outputs differed by not more than 0.82 m whereas 1.55 m and 1.32 m was realized by the six-parameter and four-parameter models. The SD values (Tables 3 and 5) estimated show a practical expression for the precision of the transformed test coordinates. Analysis of Tables 3 and 5 show that the RBFNN had the least SD values which further indicate the limit of the error bound by which every value within the transformed test dataset varies from the most probable value.
Table 5: Total error of the coordinate differences with the three methods

<table>
<thead>
<tr>
<th>PCI</th>
<th>Six parameter (m)</th>
<th>Four parameter (m)</th>
<th>RBFNN (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max Error</td>
<td>1.5525</td>
<td>1.3188</td>
<td>0.8215</td>
</tr>
<tr>
<td>Min Error</td>
<td>0.3302</td>
<td>0.3879</td>
<td>0.1894</td>
</tr>
<tr>
<td>MHE</td>
<td>1.0301</td>
<td>0.9860</td>
<td>0.4670</td>
</tr>
<tr>
<td>SD</td>
<td>0.4406</td>
<td>0.3369</td>
<td>0.2397</td>
</tr>
</tbody>
</table>

Furthermore, two statistical indicators namely, MSE and MAE were also used to ascertain the quality of the transformation results produced by the three methods. The closer these statistical indices are to zero the better the prediction capabilities of the method. With reference to Table 6, it was uncovered that the MSE and MAE values obtained for RBFNN is considerably less compared to the six-parameter and four-parameter models. The obtained results (Table 6) portray the closeness of the models fit to the measured test coordinates. On the basis of the results (Table 6), it can be seen that the RBFNN transformed plane coordinates are in better agreement with the measured coordinates compared with the six and four-parameter models. Thus, signifying the extent at which the RBFNN model could perform against an independent test data.

Table 6: Model performance statistics

<table>
<thead>
<tr>
<th>Model</th>
<th>Coordinate difference</th>
<th>PCI</th>
<th>MSE (m)</th>
<th>MAE (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 parameter</td>
<td>∆E</td>
<td>0.49374</td>
<td>0.55898</td>
<td></td>
</tr>
<tr>
<td></td>
<td>∆N</td>
<td>0.73386</td>
<td>0.72454</td>
<td></td>
</tr>
<tr>
<td>4 parameter</td>
<td>∆E</td>
<td>0.39952</td>
<td>0.5033</td>
<td></td>
</tr>
<tr>
<td></td>
<td>∆N</td>
<td>0.67001</td>
<td>0.70262</td>
<td></td>
</tr>
<tr>
<td>RBFNN</td>
<td>∆E</td>
<td>0.1159</td>
<td>0.28601</td>
<td></td>
</tr>
<tr>
<td></td>
<td>∆N</td>
<td>0.15143</td>
<td>0.32664</td>
<td></td>
</tr>
</tbody>
</table>

Moreover, to have a better idea of how well the determined parameters for the traditional techniques and the optimum RBFNN model will generalize with large unseen plane coordinates data, the whole dataset (27 common points) were used as a test data in the already determined six-parameter model, four-parameter model and optimum RBFNN model.

Figure 5 displays the horizontal errors when the whole data was used to test the determined six-parameter, four-parameter and the optimum RBFNN model. This evidently shows that within the Ghana national geodetic reference network the ANN technique of RBFNN could serve as a plausible alternative technology to be applied for plane coordinate transformation from Leigon datum to Accra datum. This claim is further confirmed by the total error attained by each method as shown in Table 7.

Table 7: Total Error of the coordinate differences for the whole data

<table>
<thead>
<tr>
<th>PCI</th>
<th>Six parameter</th>
<th>Four parameter</th>
<th>RBFNN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max Error</td>
<td>4.2158</td>
<td>4.7913</td>
<td>0.8215</td>
</tr>
<tr>
<td>Min Error</td>
<td>0.1369</td>
<td>0.1577</td>
<td>0.0283</td>
</tr>
<tr>
<td>MHE</td>
<td>1.1294</td>
<td>1.1082</td>
<td>0.2600</td>
</tr>
<tr>
<td>SD</td>
<td>0.8206</td>
<td>0.8961</td>
<td>0.2042</td>
</tr>
</tbody>
</table>

5.2 Coordinate transformation from Accra datum to Leigon datum

The derived parameters for transforming data from Accra datum to Leigon datum with their associated standard deviations using six-parameter and four-parameter transformation models are presented in Tables 8 and 9 respectively. In the RBFNN training, plane coordinates of the points in Accra datum denoted as (E\textsubscript{war}, N\textsubscript{war}) were used as the input layer neurons while (E\textsubscript{clark}, N\textsubscript{clark}) in Leigon datum were used as the output layer neurons. The MSE was used as the optimality criterion to aid in the selection of the best RBFNN structure during the training process. The optimum RBFNN structure for carrying out the transformation was [2-18-2]. That is, two inputs (E\textsubscript{war}, N\textsubscript{war}) with eighteen hidden neurons and two outputs (E\textsubscript{clark}, N\textsubscript{clark}).

Table 8: Six parameter model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>1.00000042</td>
<td>3.20E-06</td>
</tr>
<tr>
<td>b</td>
<td>1.13E-05</td>
<td>4.23E-06</td>
</tr>
<tr>
<td>c</td>
<td>-2.2234323</td>
<td>1.41753</td>
</tr>
<tr>
<td>d</td>
<td>-1.75E-05</td>
<td>3.20E-06</td>
</tr>
<tr>
<td>e</td>
<td>0.999999</td>
<td>4.23E-06</td>
</tr>
<tr>
<td>f</td>
<td>-0.50284</td>
<td>1.41753</td>
</tr>
</tbody>
</table>
Table 9: Four parameter model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>0.99999</td>
<td>2.06E-06</td>
</tr>
<tr>
<td>R</td>
<td>-1.27E-05</td>
<td>2.06E-06</td>
</tr>
<tr>
<td>ΔX</td>
<td>-2.08003</td>
<td>0.69267</td>
</tr>
<tr>
<td>ΔY</td>
<td>-2.98E+00</td>
<td>0.69267</td>
</tr>
</tbody>
</table>

Table 10 presents the extent at which the six-parameter, four-parameter and RBFNN transformed coordinates deviate from the measured plane coordinates.

Judging from the outcomes in Table 10, it can be stated that the RBFNN model is superior to the six-parameter and four-parameter models. This is because the RBFNN was able to produce more satisfactory transformed coordinates in both Eastings and Northings based on (ΔE, ΔN) as compared with the six and four-parameter models results. Again, taking into account the HE values (Table 10), it can obviously be concluded that the transformed test coordinates rendered by the RBFNN are closer to the desired testing target outputs after the testing data (untrained) were introduced to the neural network. This assertion can also be gathered from Figure 6 where the Table 10 shows the extent to which the six-parameter, four-parameter and RBFNN transformed coordinates deviate from the measured plane coordinates.

From Table 11, it can be observed that the magnitude by which the four and six-parameter models HE values deviate from the RBFNN results are very high. This signifies that if the RBFNN is applied to transform plane coordinates within the study area, better horizontal position accuracy could be achieved as compared to the four and six-parameter models. Similar phenomenon for the test point T4 estimated HE values by the six and four-parameters being better than RBFNN when transforming coordinates from Leigon to Accra datum were also observed in transforming Accra datum to Leigon datum. However, in comparison, the RBFNN produced more accurate transformed values than the six and four-parameter models.

The estimated total errors in transforming coordinates from Leigon to Accra datum are presented in Table 12. It can be seen from the analysis of the maximum and
The results presented in Table 13 shows that RBFNN transformed the plane coordinates with a better accuracy than the six-parameter and four-parameter methods in terms of MSE and MAE. These results (Table 13) show how closely the transformed coordinate are related to the observed coordinates.

Table 13: Model performance assessment

<table>
<thead>
<tr>
<th>Model</th>
<th>Coordinate difference</th>
<th>PCI</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>MSE (m)</td>
</tr>
<tr>
<td>6 parameter</td>
<td>∆E</td>
<td>0.49373</td>
</tr>
<tr>
<td></td>
<td>∆N</td>
<td>0.73385</td>
</tr>
<tr>
<td>4 parameter</td>
<td>∆E</td>
<td>0.3995</td>
</tr>
<tr>
<td></td>
<td>∆N</td>
<td>0.6700</td>
</tr>
<tr>
<td>RBFNN</td>
<td>∆E</td>
<td>0.11798</td>
</tr>
<tr>
<td></td>
<td>∆N</td>
<td>0.17782</td>
</tr>
</tbody>
</table>

Besides, to check the generalization capability of the determined six-parameter, four-parameter and the optimum RBFNN models the whole dataset was used as the testing data. On the basis of Figure 6, the horizontal error values obtained signify the extent that the horizontal transformed coordinates produced by the RBFNN, six-parameter and four parameter models deviate from the measured plane coordinates. It also shows the positional accuracy of the transformed data in horizontal terms to the measured data. In comparison, the RBFNN yielded a better horizontal positional accuracy than the six-parameter and four-parameter as illustrated in Figure 7.

Table 12: Total error of the coordinate differences with the three methods

<table>
<thead>
<tr>
<th>PCI</th>
<th>six parameter</th>
<th>Four parameter</th>
<th>RBFNN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max Error</td>
<td>1.5525</td>
<td>1.3187</td>
<td>0.7768</td>
</tr>
<tr>
<td>Min Error</td>
<td>0.3302</td>
<td>0.3879</td>
<td>0.1795</td>
</tr>
<tr>
<td>MHE</td>
<td>1.0301</td>
<td>0.9860</td>
<td>0.4997</td>
</tr>
<tr>
<td>SD</td>
<td>0.4406</td>
<td>0.3369</td>
<td>0.2320</td>
</tr>
</tbody>
</table>

The results presented in Table 13 shows that RBFNN transformed the plane coordinates with a better accuracy than the six-parameter and four-parameter methods in terms of MSE and MAE. These results (Table 13) show how closely the transformed coordinate are related to the observed coordinates.

Table 13: Model performance assessment

<table>
<thead>
<tr>
<th>Model</th>
<th>Coordinate difference</th>
<th>PCI</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>MSE (m)</td>
</tr>
<tr>
<td>6 parameter</td>
<td>∆E</td>
<td>0.49373</td>
</tr>
<tr>
<td></td>
<td>∆N</td>
<td>0.73385</td>
</tr>
<tr>
<td>4 parameter</td>
<td>∆E</td>
<td>0.3995</td>
</tr>
<tr>
<td></td>
<td>∆N</td>
<td>0.6700</td>
</tr>
<tr>
<td>RBFNN</td>
<td>∆E</td>
<td>0.11798</td>
</tr>
<tr>
<td></td>
<td>∆N</td>
<td>0.17782</td>
</tr>
</tbody>
</table>

Besides, to check the generalization capability of the determined six-parameter, four-parameter and the optimum RBFNN models the whole dataset was used as the testing data. On the basis of Figure 6, the horizontal error values obtained signify the extent that the horizontal transformed coordinates produced by the RBFNN, six-parameter and four parameter models deviate from the measured plane coordinates. It also shows the positional accuracy of the transformed data in horizontal terms to the measured data. In comparison, the RBFNN yielded a better horizontal positional accuracy than the six-parameter and four-parameter as illustrated in Figure 7.

Figure 7: Horizontal displacement of the whole data

Table 14 shows that in the case of plane coordinate transformation the RBFNN is superior to the six-parameter and four-parameter transformation models. Besides, it indicates that there is improvement of 100% in the MHE and the SD.

Table 14: Total error of the coordinate differences for the whole data

<table>
<thead>
<tr>
<th>PCI</th>
<th>6 parameter</th>
<th>4 parameter</th>
<th>RBFNN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max Error</td>
<td>4.2158</td>
<td>4.7913</td>
<td>0.7981</td>
</tr>
<tr>
<td>Min Error</td>
<td>0.1369</td>
<td>0.1577</td>
<td>0.0507</td>
</tr>
<tr>
<td>MHE</td>
<td>1.1294</td>
<td>1.1082</td>
<td>0.3389</td>
</tr>
<tr>
<td>SD</td>
<td>0.8206</td>
<td>0.8961</td>
<td>0.2362</td>
</tr>
</tbody>
</table>

6. Conclusion

Coordinate transformation is necessary in developing countries like Ghana where the local geodetic networks applied for surveying and mapping purposes are non-geocentric and highly heterogeneous in nature. This study applied for the first time the artificial neural network technology of radial basis function to transform coordinates from the two classical geodetic reference networks namely, Accra and Leigon datum in Ghana. The results obtained by using the six-parameter and four parameter transformation models confirm the existence of heterogeneity in the classical geodetic networks of Ghana. Therefore, the transformation parameters determined could not model and absorb the distortions in the coordinates between the Accra and Leigon datum. However, the application of the radial basis function neural network showed improvement in absorbing and compensating for the deformations in the classical geodetic networks in a more effective way. The conclusion to be drawn here is that, in the case of plane coordinate transformation, the artificial neural network technology could be exploited as viable alternative tool to the traditional transformation techniques applied in the present study.

Acknowledgement

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References


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