Effect of the global random distribution of data points on the estimated quality of datum transformation parameters: Simulation study

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Abstract: In this study, the global random distribution of coordinate data points was investigated for its impact on the estimated quality of the geodetic datum transformation parameters. In particular, four data sample types with different spatial distribution characteristics were simulated within a global geographical window. The clustered, purely random, random grid and regular grid patterns were investigated. In order to support and generalize the results, these four data pattern types were simulated with four common global point densities. The results showed that for all the investigated point densities, the purely random data point configuration yielded the best estimated qualities for the predicted global datum transformation parameters.

Keywords: Point patterns, Random distribution, Geodetic datums, Datum transformation parameters

1. Introduction

Since the evolution of satellite geodesy, the determination of the optimal transformation parameters among geodetic datums has been a major task. Regionally, the estimation of transformation parameters between regional geodetic systems and global geocentric ones has become a need to globalize, densify and upgrade existing regional geodetic networks (El-Tokhey, 2000; Jekeli, 2006). Also, the mutual transformation parameters among the different regional datums have become a possible task (Twigg, 2000). Globally, one of the basic tasks of the various geodetic satellite missions is the determination, upgrade and refinement of the elements of the International Terrestrial Reference Frame (ITRF). This task is accomplished via the combinations of the GNSS, SLR, VLBI and DORIS satellite techniques. In the narrow sense, such refinement encounters the determination of the global datum transformation parameters between an old ITRF and a newer one (Altamimi et al., 2011).

Most common is Burša seven parameter model which accounts for three translation components, three rotation angles about the coordinate axes and a seventh scale factor parameter (Jekeli, 2006). Irrespective of whether the datum parameter estimation is a regional or a global task, two sets of 3D-Cartesian coordinates of some common data (or control) points must be known relative to both systems (Ghilani and Wolf, 2006). Such two sets of coordinates are input as observations into a combined (or mixed) least-squares adjustment procedure, where the seven transformation parameters are the unknowns to be estimated along with their uncertainties (Leick, 1990; Ghilani and Wolf, 2006).

The quality of the input observations, the density of the control points and their spatial distribution play important role on the quality of the parameters to be estimated (Altamimi et al., 2007). The quality and density of the globally gained point coordinates are continuously being improved and their impact is positively reflected on the quality of the estimated parameters (Altamimi et al., 2013).

However, the global coordinate data points are selected to be well distributed by visual inspection only. Spatial distribution of points and their impact on the quality of the target parameters have not been explicitly studied. Practical examples of spatial distribution models are the clustered, the purely random, the randomly displaced grid and the regular grid point configurations. Such point configurations can easily be simulated and analyzed over a pre-specified rectangular geographical window, based on the principles of stochastic geometry (Stoyan et al., 1987; Stoyan, 1998; Baddeley and Turner, 2005).

For local geographical windows, these four point pattern models were used to investigate the effect of data distribution on the quality of digital elevation models (Hassouna, 2013a) and on the quality of geoidal height prediction (Hassouna, 2013b). The results of the two studies implied that the randomness of data point configurations might be competitive to their regularity, in yielding better prediction qualities. In the second work, it was emphasized that the data distribution impact could vary according to the investigated geodetic task. So, it was recommended that the effect of data distribution should be separately studied for each task (Hassouna, 2013b).

Preliminary investigations have revealed that the spatial distribution variation of data point samples, within limited regional geographical windows like Egypt, would not significantly affect the quality of the target global transformation parameters. Whatever the point patterns, such limited windows would yield practically similar sparse global data coverage, thus resulting in negligible variations in the estimated qualities of the global parameters.

Thus, the objective of the current study is to investigate the effect of the global random distribution of the data points on the quality of the estimated datum transformation parameters. In particular, the aforementioned four point pattern types will be simulated to represent different sets of coordinate data points over a global window. This proposed simulation algorithm stands in analogy with the optimization of geodetic networks configurations before the collection of actual observations (Ashkenazi and Cross, 1972).

2. The simulated point configurations

In particular, four types of global point configurations were simulated, utilizing the software developed by Stoyan (1998). These are the Matérn-clustered pattern, the Poisson's purely random pattern, the random grid and the regular grid (Schmidt, 2011). The random displaced grid was simulated with a spatial standard deviation of 0.2 arc-degree (Stoyan, 1998). In order to support and generalize the results, all point pattern types were simulated with four numbers of global data points. Each type was generated with an average of 20, 80, 120 and 155 points. Such numbers of points were arbitrarily selected in order to verify the impact of data distributions on the quality of estimated parameters, taking into account the variation in global data density. Also, a global number of 120 or 155 points might be realistic for a set of ITRF stations.

In Hassouna (2013b), it was found that different realizations of the same point pattern type yielded practically the same spatial distribution characteristics. This agrees with the Ergodizity property of spatial point processes (Stoyan and Stoyan, 1992; Schmidt, 2011). So, in the current study, for a given global point density, only one point pattern type was simulated. Having geodetic latitudes and longitudes as horizontal curvilinear coordinates, these sixteen point configurations were realized within a rectangular global window bounded by ($-90° \leq \varphi \leq +90°$; $-180° \leq \lambda \leq 180°$). As illustrative examples, Figure (1) shows the four point pattern types with 120 points.

The clustered sample is irregular. In a purely random pattern, the point locations are spatially independent (Stoyan and Penttinen, 2000; Schmidt, 2011). Such sample is more regular than a clustered one. The randomly displaced grid is more uniform than the previous two types, whereas it is less regular than a grid pattern. However, it still exhibits slight randomness. Among all types, the regular grid pattern is the most uniform one. Statistically, both the clustered and regular grid patterns can be looked upon as exhibiting two opposite extreme systematic point arrangements (Ohser and Lorz, 1996).

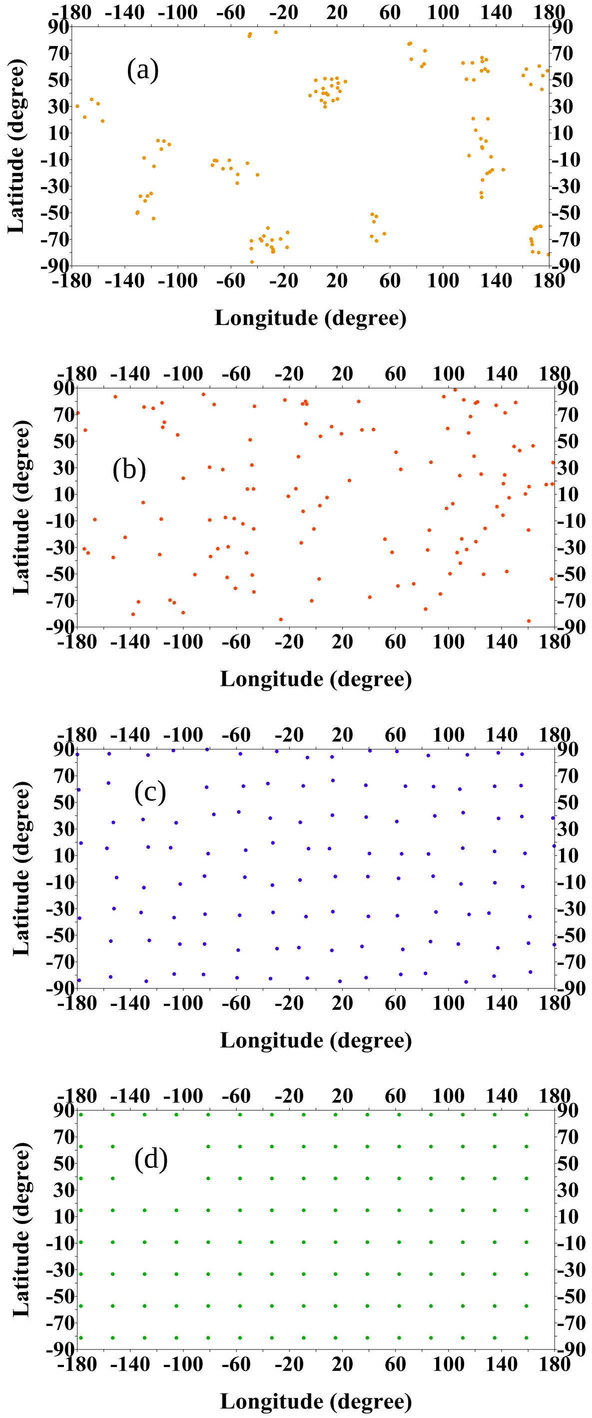

Figure 1: Illustrative point pattern types with 120 points (a) Clustered pattern (CT); (b) Purely random pattern (RN); (c) Random grid pattern (RG); and (d) Regular grid pattern (GR)

3. The simulated sets of noisy Cartesian coordinates

The above simulated sets of horizontal geodetic coordinates were assumed to be relative to the WGS-84 reference ellipsoid. In order to devote the current study to the effect of horizontal data distributions on the datum parameters quality, such simulated points were assigned zero ellipsoidal height values. The numerical significance of non-zero ellipsoidal heights will be clarified in Section (4). Thus, using the geodetic curvilinear coordinates (φ, λ, h=0) of the points of the sixteen simulated configurations, the corresponding 3D-Cartesian ones were computed by (Jekeli, 2006)

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} (R_n + h)\ \cos\varphi\ \ \cos\lambda \\ (R_n + h)\ \cos\varphi\ \ \sin\lambda \\ \left[(b^2/a^2) R_n + h\right]\ \sin\varphi \end{bmatrix}, \qquad (1)$$

where a and b are semi-major and semi-minor axis of the WGS-84 reference ellipsoid, respectively; and R_n is the prime-vertical radius of curvature at the point under consideration. For WGS-84, a = 6,378,137.0 and b = 6,356,752.3142 meters (Ghilani and Wolf, 2006).

The above computed sets of Cartesian coordinates were considered to represent start input coordinates, from which respective start sets of Cartesian coordinates were evaluated relative to another arbitrary coordinate system. The seven transformation parameters from the WGS-84 system to that arbitrary system were assigned the following adopted values

$$\text{Shift components} \equiv \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} = \begin{bmatrix} +150 \\ +115 \\ +7 \end{bmatrix} \text{ meter},$$

$$\text{Rotation angles} \equiv \begin{bmatrix} R_X \\ R_Y \\ R_Z \end{bmatrix} = \begin{bmatrix} +3 \\ +2 \\ +3 \end{bmatrix} \text{ arcsec.},$$

$$\text{Scale factor } S = 0.2 \text{ ppm.} \qquad (2)$$

The rotation angles about the coordinate axes are taken positive anti-clockwise. Hence, the sixteen sets of start point coordinates, which correspond to the second coordinate system, were assessed as follows (Jekeli, 2006)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} + (1+S) \begin{pmatrix} 1 & R_Z & -R_Y \\ -R_Z & 1 & R_X \\ R_Y & -R_X & 1 \end{pmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}, \qquad (3)$$

where the small rotation angles are expressed in radian.

From Eq. (3), it is apparent that such adopted parameters (in Eq. 2) are consistent with both sets of Cartesian coordinates. Of course, any known regional or global datum parameters could be used instead, since all such datums are differentially similar.

It is worth mentioning that the transformed point configurations, as computed from Eq. (3), would practically show the same spatial distributions in terms of geodetic latitudes and longitudes, as those input to Eq. (1). This can be supported by the intuition that such two nearly aligned geodetic datums would yield systematic changes in the respective geodetic latitudes and longitudes, which are surely in range of arc-seconds.

As input simulated observations, both sets of the start Cartesian coordinates, in Eqs. (1) and (3) were assigned Gaussian normally distributed noises with a zero mean and a standard deviation of one meter (Press et al., 2001). Such unification of the simulated noise was thought to help in rendering the quality of the subsequently estimated datum parameters to the effect of data distribution, rather than to the relative quality of observations.

Then, for each of the sixteen "noisy" pairs of point configurations, a mixed least squares adjustment was performed, yielding the respective sets of seven transformation parameters along with their uncertainties.

4. The mixed least-squares adjustment model

The mixed least-squares adjustment functional model can be expressed as follows (Mikhail and Gordon, 1981)

$$F_{c,1}\left(\bar{P}_{u,1}\ ,\ \bar{L}_{n,1}\right) = 0, \qquad (4a)$$

where

- $\bar{P}_{u,1}$ the vector of adjusted parameters,
- $\bar{L}_{n,1}$ the vector of adjusted observations,
- u the number of parameters,
- n the number of observations,
- c the number of condition equations among the adjusted parameters and observations.

Based on Eq. (3), for each coordinate data point common to both datums, the mathematical model in Eq. (4a) can be written as follows (El-Tokhey, 2000; Ghilani and Wolf, 2006)

$$\begin{bmatrix} \bar{X}_0 \\ \bar{Y}_0 \\ \bar{Z}_0 \end{bmatrix} + (1+\bar{S}) \begin{pmatrix} 1 & \bar{R}_Z & -\bar{R}_Y \\ -\bar{R}_Z & 1 & \bar{R}_X \\ \bar{R}_Y & -\bar{R}_X & 1 \end{pmatrix} \begin{bmatrix} \bar{X} \\ \bar{Y} \\ \bar{Z} \end{bmatrix} - \begin{bmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{bmatrix} = 0. \qquad (4b)$$

Thus, for a pair of point configurations having K points, the vectors of adjusted parameters and observations are given by

$$\overline{P}_{u,1} = \begin{bmatrix} \overline{X}_0 \\ \overline{Y}_0 \\ \overline{Z}_0 \\ \\ \overline{R}_X \\ \overline{R}_Y \\ \overline{R}_Z \\ \\ \overline{S} \end{bmatrix}, \quad \overline{L}_{n,1} = \begin{bmatrix} \overline{X}_1 \\ \overline{Y}_1 \\ \overline{Z}_1 \\ \overline{x}_1 \\ \overline{y}_1 \\ z_1 \\ \cdots \\ \cdots \\ \overline{X}_k \\ \overline{Y}_k \\ \overline{Z}_k \\ \overline{x}_k \\ \overline{y}_k \\ z_k \end{bmatrix}. \quad (5)$$

Thus, u = 7, c = 3K and the degree freedom of the mixed adjustment process is given by

$$\begin{aligned} r &= c - u \\ &= 3K - 7. \end{aligned} \quad (6)$$

The corresponding linearized form of the mixed adjustment algorithm was formulated as follows (Mikhail and Gordon, 1981)

$$A_{c,u}\ \delta_{u,1} + B_{c,n}\ V_{n,1} = W_{c,1}\ , \quad (7)$$

where

$A_{c,u}$ the design matrix, which carries the partial derivatives of the functional model (Eq. 4a) with respect to the parameters,

$\delta_{u,1}$ the vector of parameters corrections,

$B_{c,n}$ the coefficient matrix of condition equations, which contains the partial derivatives of the functional model (Eq. 4a) with respect to the observations,

$V_{n,1}$ the vector of observations residuals,

$W_{c,1}$ the vector of absolute terms.

While the partial derivatives in the matrix A are functions of the observations, those in B are expressed in terms of the approximate parameters (Ghilani and Wolf, 2006).

As stated in Section 3, the input observations were assigned equal noise standard errors. So, the least-squares adjustment algorithm was performed using equal weights. Thus, using an a priori reference variance of unity, the solution vector was computed as follows (Leick, 1990)

$$\delta = \left[A^T \left(BB^T \right)^{-1} A \right]^{-1} \left[A^T \left(BB^T \right)^{-1} W \right]. \quad (8)$$

Then, the observational residuals were assessed by

$$V = B^T \left(BB^T \right)^{-1} \left[A\,\delta - W \right]. \quad (9)$$

The seven datum parameters were initially assigned zero approximate values. The solution was improved, using the updated values of the observations and parameters, till the convergence of iterations. Denoting the approximate parameters of the last iteration as

$$\left[X_0^{\,a}\ Y_0^{\,a}\ Z_0^{\,a} \quad R_X^{\,a}\ R_Y^{\,a}\ R_Z^{\,a} \quad S^{\,a} \right]^T,$$

and the corresponding estimated corrections as

$$\left[\delta X_0\ \ \delta Y_0\ \ \delta Z_0 \quad \delta R_X\ \ \delta R_Y\ \ \delta R_Z \quad \delta S \right]^T,$$

so, the estimated datum parameters were assessed as follows

$$\begin{bmatrix} \overline{X}_0 \\ \overline{Y}_0 \\ \overline{Z}_0 \\ \\ \overline{R}_X \\ \overline{R}_Y \\ \overline{R}_Z \\ \\ \overline{S} \end{bmatrix} = \begin{bmatrix} X_0^{\,a} \\ Y_0^{\,a} \\ Z_0^{\,a} \\ \\ R_X^{\,a} \\ R_Y^{\,a} \\ R_Z^{\,a} \\ \\ S^{\,a} \end{bmatrix} + \begin{bmatrix} \delta X_0 \\ \delta Y_0 \\ \delta Z_0 \\ \\ \delta R_X \\ \delta R_Y \\ \delta R_Z \\ \\ \delta S \end{bmatrix}. \quad (10)$$

And the reference variance was computed by

$$\hat{\sigma}^2_0 = V^T V \big/ r. \quad (11)$$

Finally, the covariance matrix of the estimated seven parameters was assessed as follows

$$\Sigma_{PP} = \hat{\sigma}^2_0 \left[A^T \left(BB^T \right)^{-1} A \right]^{-1}. \quad (12)$$

Considering Eqs. (1), (4b) and (7), it could be sensed that even if the ellipsoidal heights were not ignored, they would have negligible numerical impact on the computed Cartesian coordinates. This numerical insignificance is reflected on the elements of the matrix A, and consequently, on the estimated covariance matrix (Eq. 12). Such thought could be verified by keeping in mind that those heights, even if they amounted to a few kilometers, are very small compared to R_n, which represents the principal maximal radius of curvature at any point. Perhaps exceptions could arise for points that happen to have so large ellipsoidal heights and are located at the equator or at the poles.

In the current work, the above steps of the mixed least-squares procedure (Eq. 6 to 12) were performed using the DATUM software developed by Junkins (1998).

In Section 5, both the local and global precision criteria for the estimated seven parameters are explored and compared for all the investigated data point samples. As local measures of precision, the estimated standard errors of the parameters will be used. On the other hand, the traces of the estimated covariance matrices will be used as global precision measures (Blaha, 1971; Ashkenazi, 1974). For this purpose, it was necessary to have consistent sums for the diagonal elements with units of m^2. Using a mean Earth's radius of 6371 km, the standard errors of the rotation angles were converted to respective axes-displacement uncertainties on the Earth's surface in meter units as follows (Altamini et al., 2007)

$$\begin{bmatrix}\sigma_{\bar{R}_X}\\ \sigma_{\bar{R}_Y}\\ \sigma_{\bar{R}_Z}\end{bmatrix}_{meter} = \left(\frac{6371000}{206264.8}\right)\begin{bmatrix}\sigma_{\bar{R}_X}\\ \sigma_{\bar{R}_Y}\\ \sigma_{\bar{R}_Z}\end{bmatrix}_{arc\,sec.} , \quad (13)$$

and the estimated standard errors of the scale factors were expressed as radial errors by (Altamini et al., 2007)

$$\sigma_{\bar{S}\ meter} = \left(\frac{6371000}{10^6}\right)\sigma_{\bar{S}\ ppm}. \quad (14)$$

5. Results

Figure (2) plots the estimated reference variances which correspond to the four point pattern types with global number of points K= 20, 80 and 120. Figure (3) shows the respective estimated standard errors of the three datum shift components, while Figure (4) illustrates the estimated uncertainties of the three datum rotation angles. Also, Figure (5) plots the corresponding standard errors of the estimated scale factors.

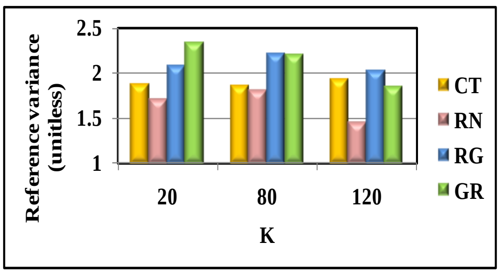

Figure 2: The estimated reference variances

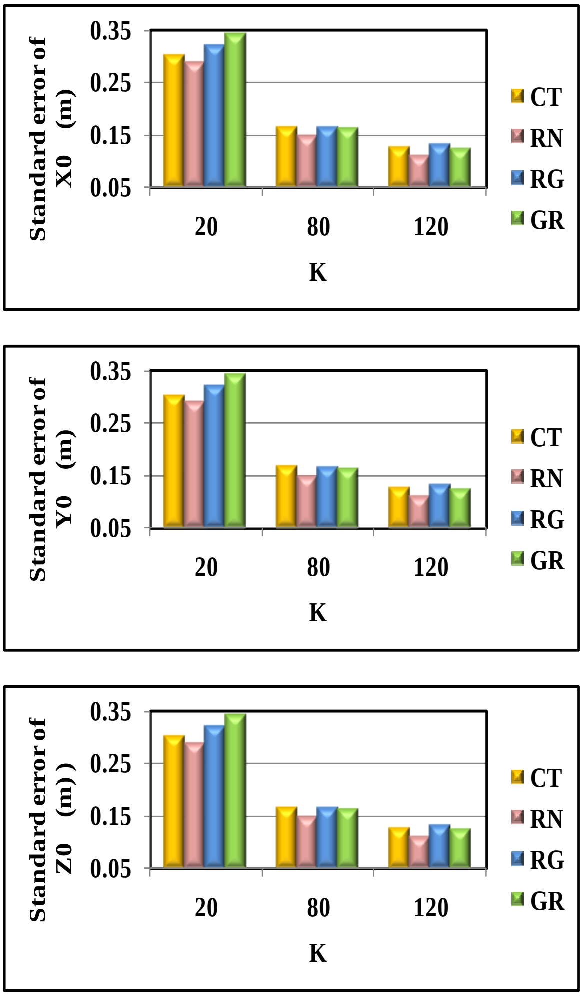

Figure 3: The estimated standard errors (top) of X_0 components, (middle) y_0 components; and (bottom) the Z_0 components

Figures (2 - 5) show that whatever the global data point density is, the pure random data point configuration resulted in minimal values of reference variances and optimal uncertainties of the estimated seven datum transformation parameters. Also, Figures (3 - 5) show significant differences among the uncertainties relevant to the pure random point sample type, and those corresponding to the other three configuration types. The significance of such differences can be directly sensed for the shift components. The significant differences among the uncertainties of the rotation angles and scale could be better sensed, if they are expressed in meter units on the Earth's surface and in the radial direction, respectively (Eqs. 13 and 14). Figure (3) shows that for a specific data configuration, the uncertainties of the shift components are nearly the same. This can be regarded to the fact that the mathematical model (in Eq. 4b) is marginally linear with respect to the shift parameters (Ghilani and Wolf, 2006).

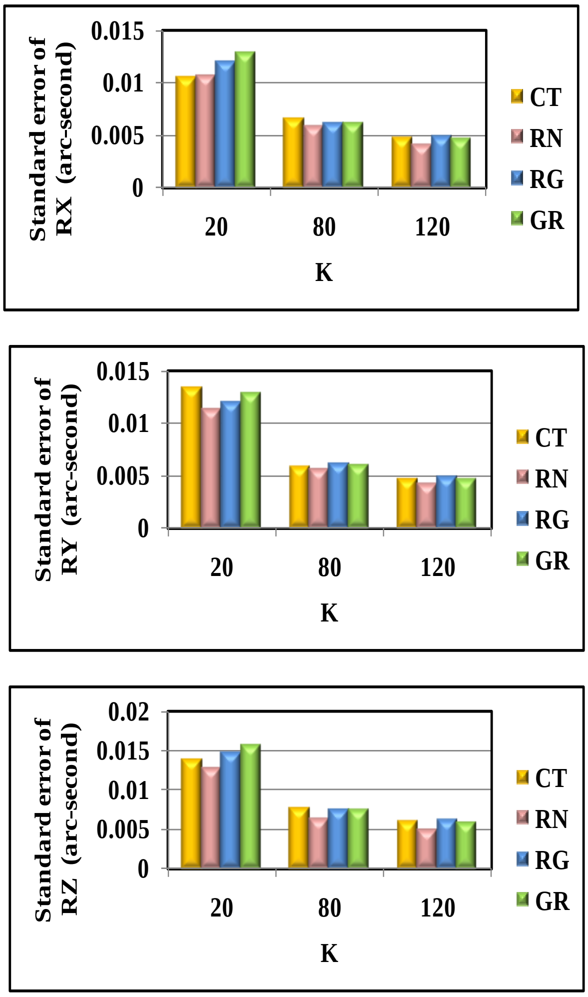

Figure 4: The estimated standard errors of (top) the R_x rotations; (middle) the R_y rotations; and (bottom) the R_Z rotations

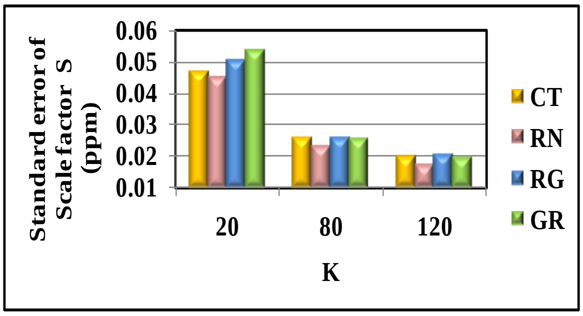

Figure 5: The estimated standard errors of the scale factors S

As the observations were assigned similar simulated noise, the minimal values of the variance factors imply respective optimal norms of the observational residuals that were produced by the adjustment process. In other words, the random point configurations yielded optimally filtered observational noise during the adjustment. On the other hand, the optimal standard errors pertaining to the random pattern type (Figures 3 - 5) imply respective minimal traces of the variance-covariance matrices (Eq. 12). This in turn means that the normal matrices relevant to the random point configurations are the most well conditioned ones, compared to those of the other data point patterns (Blaha, 1971; Ashkenazi, 1974). Such results imply that the random data point samples lead to optimal error propagation characteristics through the design matrix A. Namely, as denoted in Section 4, such A matrix depends on the observations which are point coordinates in the current investigation. Such optimal normal matrix conditioning must have been reflected on the respective estimated parameters (Eq. 10) and their uncertainties (Eq. 12).

Figures (6 - 9) plot similar comparisons, but only regarding the four point pattern types with K=155 points. Again, Figures (6 - 9) show that the random point samples yielded optimal variance factors and datum parameters uncertainties. Furthermore, the differences among the various quality measures are still significant.

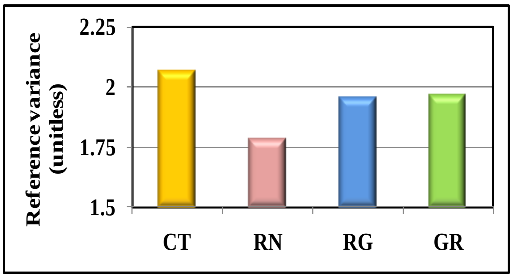

Figure 6: The estimated reference variances (K=155)

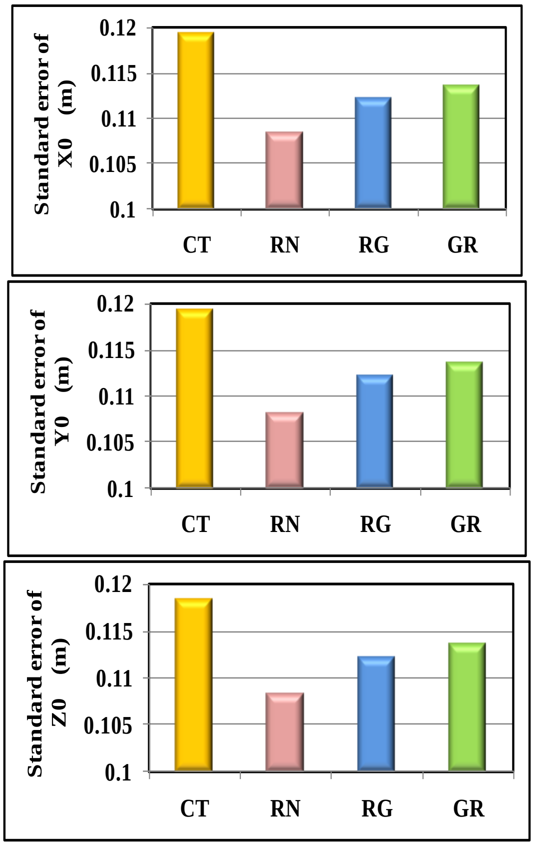

Figure 7: The estimated standard errors of (top) the X_0 components; (middle) the Y_0 components; and (bottom) the Z_0 components (K=155)

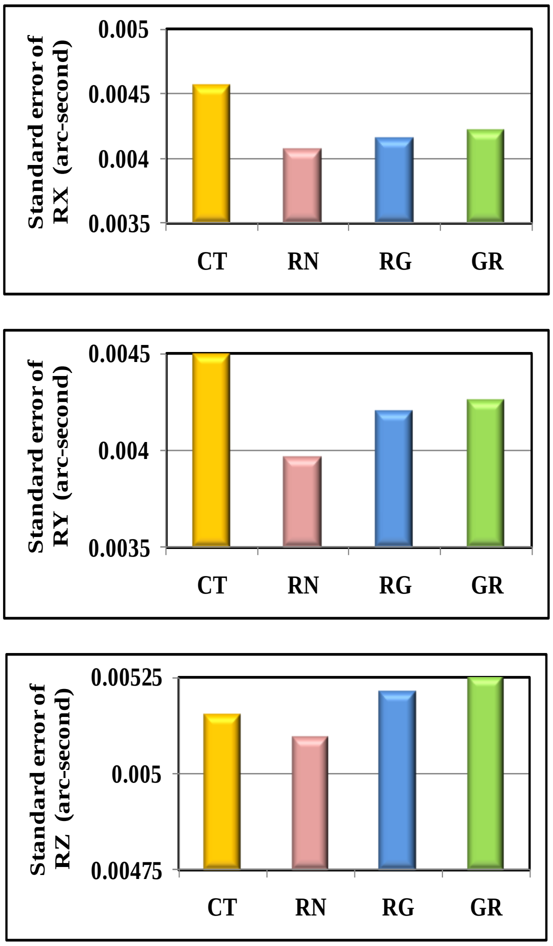

Figure 8: The estimated standard errors of the (top) R_x rotations; (middle) R_y rotations; and (bottom) R_Z rotations

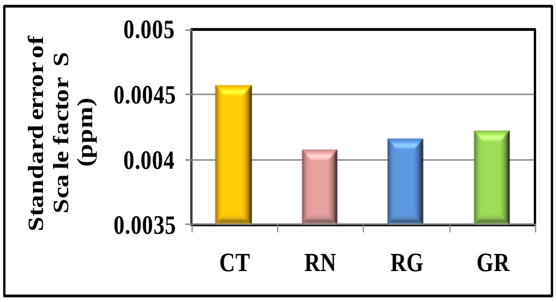

Figure 9: The estimated standard errors of the scale factors S (K=155)

Again, Figure (10) assures that, among the other three types, the purely random point configurations yielded the optimal global-wise precision for the estimated parameters. So, this final result supplements and is mutually related to the local-wise optimality of the purely random point samples in Figures (3 - 5) and Figures (7 - 9).

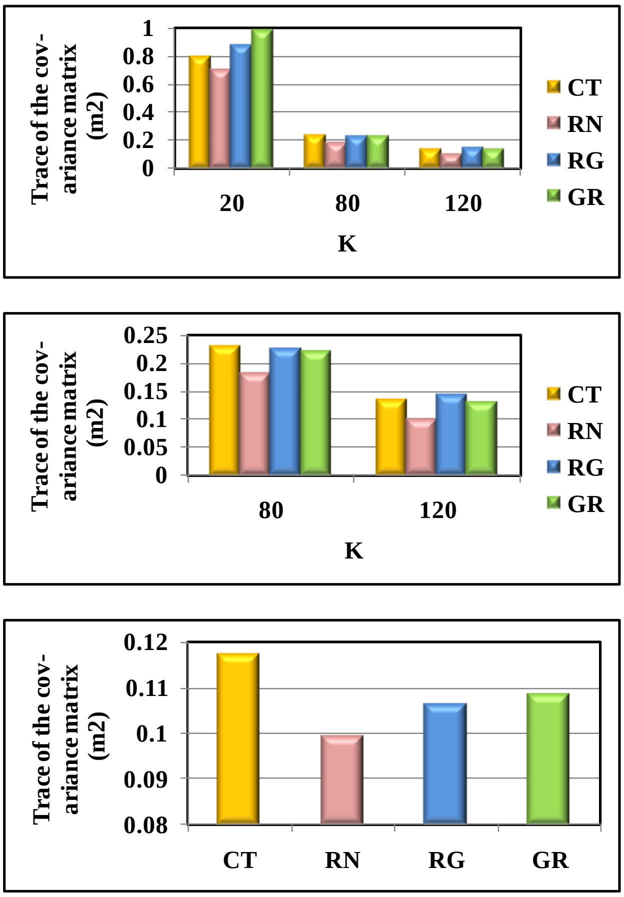

Figure 10: Comparison among the trace values of covariance matrix (top) for k = 20, 80 and 120; (middle) for k = 80 and 120; and (bottom) (K=155)

6. Discussions and recommendations

For the four investigated data point densities, the purely random coordinate data distribution proved to be the optimal one. This result was ascertained, based on both the local and global parameter precision criteria. Although the followed mixed adjustment procedure is originally regular, the minimal trace proximity of the random samples could stand in analogy with the free adjustment of geodetic networks (Blaha, 1971; Ashkenazi, 1974). So, among any other distributions of the data samples, the random point configurations could be considered as offering the optimal numerical stability of the normal matrix (Blaha, 1971). This was associated with mostly well conditioned normal matrices, which yielded optimal solutions and uncertainties for the respective estimated datum parameters.

Regarding the clustered (CT), nearly regular (RG) and regular (GR) configurations, the corresponding systematic arrangements of data points (Ohser and Lorz, 1996) could have worsen the condition number of the normal matrices, which negatively affected the estimated parameters qualities.

Hence, regarding the estimated parameters and their qualities, the purely random point patterns could be considered to have minimized the numerical problems, due to the arbitrary selection of a data distribution pattern. In this sense, the qualities of the adjusted parameters might truly reflect the precision of the input observations.

Consider again the optionally adopted datum parameters (Eq. 2). The obtained results depict the major impact of the observational data on the estimated qualities of datum parameters. So, such results also assure that the same quality proportions could have been obtained, if another known or arbitrary set of parameters was adopted in the simulation process.

In comparison with the current realistic coordinate data noise and transformation parameters relevant to the successive ITRFs, the simulated input observational noise and the adopted magnitudes of the datum parameters are considerably large. Also, this holds true for the estimated quality measures (Altamimi et al., 2011). Such larger proportions were thought to help compare the variations of the parameter qualities with data distribution. Moreover, the investigation of actual ITRFs coordinate data distributions would surely yield significant differences among the output quality measures. Such quality differences, although might be much smaller, would be significant, compared to the nowadays strict ITRFs demands. For example, the adverse effect of a poor ITRF data coverage over the southern hemi-sphere, was apparent on the quality of the Z_0 parameter between ITRF2005 and ITRF2008 (Altamimi et al., 2011).

So, it is recommended to optimize the actual ITRF coordinate data distributions, regarding the estimated ITRF transformation parameters, their time rates and their uncertainties. Such data distribution optimization could be accomplished, leaning on the stochastic geometry approach, which was followed in the current study. It might be claimed that the investigated data distribution models might be only of theoretical benefit. However, based on the principles of stochastic geometry, there exist efficient computer modules that could statistically judge, whether a given proposed data pattern belongs to a specific spatial distribution model (e.g. Baddeley and Turner, 2005). Also, as the majority of the ITRFs stations are mostly continent- or nation-wise clustered (Altamimi et al., 2007 & 2011), such future outlook could be of benefit.

Regarding positioning by GNSS, it is well known that the quality of the estimated positions and position differences is highly dependent on the geometry of the tracked satellite constellation. Such geometry is often evaluated by the Dilution Of Precision (DOP) factors (Leick, 1990). Namely, the DOP factors are routinely used as an optimization criterion for the planning and post-processed solutions of the GNSS sessions. Again, a future work could be to try optimizing the GNSS observational sessions, leaning on specific spatial distribution models for the GNSS satellites. This is easy to achieve, based on the geocentric curvilinear coordinates of the GNSS satellites. So, using appropriate stochastic geometry modules, software packages could be designed and tested for the planning and post-processing of the GNSS observational sessions.

Finally, the current study could help optimizing local data configurations for country-level datum parameter estimation. However, in such case, the Molodensky-Badekas model might be used (Ng Boon Chye, 1992). This ten (7+3) parameter model accounts for the rotations and scale with respect to a central (average) origin that is close to the data points, and whose local 3D-coordinates represent the three additional parameters (Boon and Setan, 2007). In this manner, the problem of poor global coverage of the local data patterns, which was declared in Section 1, might be abridged.

7. Conclusions

Based on the current study, it can be concluded that the global density of the coordinate data points affects the absolute magnitudes of the estimated precisions of the datum parameters. However, the relative magnitudes of quality are dependent on the spatial distribution of the coordinated data samples. The current study could help optimizing local data configurations for country-level datum parameter estimation. The ten parameter model accounts for the rotations and scale with respect to a central (average) origin that is close to the data points, and whose local 3D-coordinates represent the three additional parameters. In this manner, the problem of poor global coverage of the local data patterns can be overcome considerably.

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