Journal of Geomatics Vol 11 No. 2 October 2017

Comparative analysis of the standard error in relative GNSS positioning for short, medium and long baselines

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(Received: Jan 21, 2017; in final form: Oct 12, 2017)

Abstract: In GNSS Relative positioning, both in post processing and real time positioning, GNSS baseline processing plays a vital role in determining the standard error in the positioning. This paper presents a comparative analysis of the standard error in relative GNSS positioning for short, medium and long baselines. Satellite observations were acquired on 19 control points within the campus of university of Lagos, using both the passive and active (CORS) station principles in Post Processing GNSS positioning data at differential mode. The short baselines have maximum length not exceeding 1.5km from the control points to a base station in university of Lagos, the medium baselines have range not exceeding 12km from the control points to a CORS located at the Lagos state Surveyor General's Office, Nigeria, while the long baselines have range not exceeding 107km from the control points to another CORS (Continuously Operating Reference Station) located in Cotonuo, Benin Republic. After post processing operation, the standard error in relative position was computed from the satellite geometry model. The results were statistically analysed using ANOVA One Way at 0.05 significant level comparing the average standard error in relative positioning of all stations during the three baseline observation scenario. Subsequently, Scheffe test was conducted on the ANOVA results. The statistical results show no significant difference between the level of standard error obtained by the baseline processing involving the CORS at 12km and the Base receivers at 1.5km but there exists a significant difference between the 107km CORS baseline processing and the 12km as well as the 1.5km baselines. The short baselines were found to have the highest achievable processing precision while the long baselines have the least. The study shows that the longer the baselines the lower the standard error in relative positioning even with CORS. This however, does not negate the reliability of the long baseline results but defines the level of precision and accuracy achievable when compared with other baseline length with a view to further more researches on long baseline error reduction.

Keywords: Baseline, Standard Error, Global Positioning System (GPS), Sky plot, Dilution of Precision (DOP)

1. Introduction

The emergence of Global Positioning System (GPS) has revolutionized the process of position determination and the navigation techniques. GPS is a satellite positioning system based on one-way ranging in which the measurement of travel time of a signal from transmitter to receiver is achieved by the application of separate clocks; the transmitter (GPS satellites in space) and the receiver clocks (GPS receivers on the earth's surface). The two clocks must be properly synchronized as a deviation of one nanosecond is equivalent to 30cm in distance (Rizos, 1999).

Before the development of the GPS various positioning system had been used in fixing the point position on the earth surface. Most simple and widely used method was 'traversing' which involved a series of connected lines whose distances and bearings are known. Omogunloye (1988) presented a methodology for determination of optimal number of stations between Azimuth checks in a traverse network. The least square regression approach was applied by him to determine the optimal number of stations. Other methods used were triangulation and trilateration. These methods can be applied to network of control determination and adjustment. Omogunloye (2010) presented a method of simulated annealing for the optimal adjustment of the Nigerian Horizontal Geodetic Network adjustment. The method of least square adjustment was also used in the simulated annealing. Omogunloye (1991), presented various field methods and quality control in geophysical prospecting.

The principle of GPS positioning is based on the measurement of the ranges between the receiver placed at unknown positions and a few simultaneously observed satellites. The positions of the satellites are forecasted and broadcasted along with the GPS signal to the user. Through several known positions (of the satellites) and the measured distances between the receiver and the satellites, the position of the receiver can be determined (Xu, 2007).

The Differential Global Positioning System (DGPS) involves position determination of a rover station with reference to a base station. Both the rover and base stations simultaneously observe the same GPS satellites in space and necessary pseudo-range correction is applied on the position of the rover station with respect to the base station which could be post processed or real time by radio transmission. DGPS positioning could either be in static or in Kinematic mode. The purpose of Differential correction in DGPS positioning is to provide a higher accuracy in GPS position determination which is not achievable in Precise Point Positioning (PPP) DGPS positioning has applications in various field such as in dynamic offshore positioning for oil exploration, where it serves as the positioning reference system, in construction industry, all forms of mapping activities, deformation monitoring, etc.

Furthermore, other satellite constellations beside the GPS have been developed and still in development; the Russian GLONASS, the European Galileo, the Chinese BeiDuo/COMPASS and the Japanese QZSS. Currently, there are three GNSS constellations that are fully operational (GPS, GLONASS, and QZSS) and two that are being actively deployed (COMPASS and Galileo). These have increased the number of available satellites and it is still increasing with the introduction of new and modernized satellite constellations. (Trimble, 2012)

The combination of these system in satellite based positioning have given rise to GNSS and now areas that were previously too obscured could be reached with modern GNSS rover. These multiple navigation systems operating independently help increase the awareness and accuracy of the real time positioning and navigation. A combined GNSS system which uses the GPS, GLONASS and Galileo systems together has a constellation of about 75 satellites. A constellation of 75 satellites increases satellite visibility of GNSS receivers especially in urban canyons (Xu, 2007).

GNSS technology has further more research in satellite based positioning system. The principle of operation of GPS in position determination has not changed in GNSS but an expectation of achieving greater accuracy and precision with GNSS is envisage. Baseline processing, the fundamental principle of satellite based positioning is still applicable with the GNSS system both in static and differential mode. The baselines spans from short to long ranges with various error compensation and correction applied to longer baseline to achieve desired precision and accuracy with the use of various commercial GNSS data processing software.

The Global Navigation Satellite System has dramatically changed the way that surveyors and other professional engineers measure positional coordinates. These experts can now measure spatial distances – baselines and estimate 3D coordinates of a new point (rover) relative to a reference located from a few to many tens of kilometers away (Fotiou et al., 2006).

This range/baseline defined by the distance between the rover and the base station is a position vector whose origin is at the base station. Thus, the position vector of the rover station defines the DGNSS baseline (range vector). In DGNSS positioning, the increase in the baseline affects the accuracy of the determined position and this accuracy is also a function of the satellite geometry. It is also worthwhile to note that satellite geometry has an amplifying effect on other GNSS sources of error (Lonchay, 2009).

Recent development in GNSS has led to a paradigm shift from passive network of geodetic controls to active CORS. The active stations are continuously developed into a network system capable of reducing the number of stations over a coverage area by extending baseline length and at the same time improving the accuracy of processing the baselines between the reference stations and the rovers. This could be achieved either from a networked GNSS stations where all stations are linked to a central control station for data correction and modelling or the most advanced technique nowadays based on the VRS network concept (Retscher, 2011).

Looking at the recent development in GNSS and CORS the study is aimed at carrying out a comparative analysis of the standard error in relative positioning for short, medium and long baselines in GNSS positioning. This was achieved by acquiring positional data using a GNSS receiver on 19 selected control stations within University of Lagos Nigeria, processing the observations with respect to a mounted conventional base station within the university forming the short baseline, a CORS (Continuously Operating Reference Station) in Lagos State located in the Lagos State Surveyor General's office over 10km away from the university (forming the medium baseline) and a CORS in Cotonou, Benin Republic over 100km away (forming the long baseline). The standard error in relative positioning was computed with reference made the Positional Dilution of Precision (PDOP) and the standard error in range measurements. These two factors are functions of the satellite geometry. The results were subsequently analysed and presented.

2. The satellite geometry

The nature of the GPS satellite constellation is of particular interest when considering the use of the system to determine height. The constellation consists of at least 24 operational satellites, which are divided into 6 orbital planes evenly spaced about the equatorial plane (Hamish, 2004).

The orbital planes contain 4 satellites that are inclined at 55° with respect to the equatorial plane. As a result, the satellites that are visible to the observer are a function of both the 55° inclination of the satellite orbital planes and the observer's latitude (Hamish, 2004).

For instance, an observer at latitude 90° south cannot view any satellites above a 45° elevation mask due to the 55° orbital inclination (see Figure 1a). Conversely, an observer at latitude 45° south cannot satellites in a southern direction except at elevations very close to the zenith (Figure1b) (Hamish, 2004)

The geometry of satellites, or lack of it, has obvious implications with regard to positioning. If one wishes to attain a reliable vertical solution, the geometry of the satellites being observed is critical. As with terrestrial resections, a well-defined solution requires a good geometrical spread of control stations about the unknown point. In the case of a GPS derived position there are no satellites available below the horizon. This induces a bias into the vertical component making height determination less precise than horizontal (Hamish, 2004).

Figures 1a and 1b highlight the problems faced by those wishing to make GPS observations to determine precise height. When making observations at 90° south the solution is weakened by the lack of satellites towards the zenith while at 45° south the solution is weakened by the lack of satellites in the southern direction. When making observations over a prolonged period, such as 24 hours, many satellites rise and set. Accordingly, geometry does not play the same role as it may if one were undertaking observations over a shorter duration (Hamish, 2004).

3. Dilution of Precision (DOP)

If one considers that the design matrix needed to construct the normal equations for a least squares solution, in addition to the systematic errors of the observations, is a function of the satellite observation direction then it is clear that satellite sky distribution plays an important part in the propagation of errors with respect to unknown parameters (Santerre, 1991).

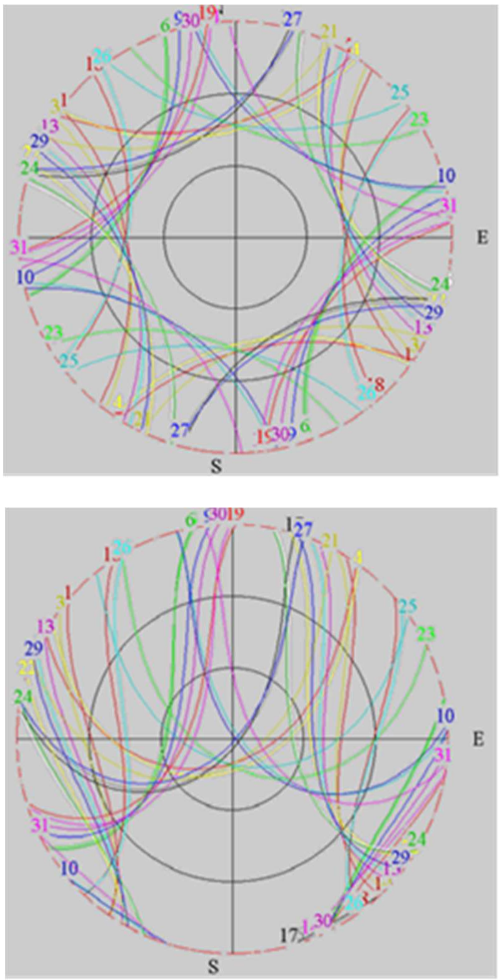

Figure 1a: Sky plot of visible satellites at (a-top) 90°south; (b-bottom) 45°south
(Source: Hamish, 2004)

The DOP factors are derived from the inverse of the unweighted normal equation matrix used to determine position and as such are strictly geometrical indicators of satellite suitability for positioning. The GDOP, PDOP and TDOP are determined from the cartesian coordinates in the World Geodetic Reference System 1984 (WGS84) while the HDOP and VDOP factors are derived from the transformed horizontal and vertical components in terms of the local system being used (Hofmann-Wellenhof et al., 2001).

DOP is an indicator of the quality of the geometry of the satellite constellation. Your computed position can vary depending on which satellites you use for the measurement. Different satellite geometries can magnify or lessen the errors in the error budget described above. A greater angle between the satellites lowers the DOP, and provides a better measurement. A higher DOP indicates poor satellite geometry, and an inferior measurement configuration (Corvallis, 2000)

4. Study area

The research was carried out on some selected control points within University of Lagos, Lagos State Nigeria with reference made to a conventional base station on a first order control within the campus, a CORS located at the Office of the Surveyor General of Lagos State and the other CORS located in Littoral State of Benin Republic, Cotonuo. Nigeria lies between Longitudes 30 E and 140 E and Latitude 40 and 140 N (Figure 2). Lagos State in Nigeria lies between Longitude 2º 45'E to 4º 20'E and Latitude 6º 2'N to 6º 27'N. Benin Republic lies between Longitude 1º E to 3º 40' E and Latitude 6º 30' N to 12º 30' N while Cotonuo In Benin republic lies between Longitude 2º 26' E and Latitude 6º 22'N

Figure 2: Imagery showing the map of Nigeria and Benin republic with the Location of Cotonou, Benin Republic and Lagos, Nigeria. (Source:Google earth)

5. Instrumentation

Trimble R5 GNSS receiver was used in acquiring the satellite ephemeris at the selected control stations within University of Lagos. The data acquired was processed with respect to data acquired by another Trimble R5 GNSS receiver located at XST347 base station to form the short baseline. Simultaneous observation data set were also downloaded from the Lagos CORS and the Cotonou CORS for medium and long baseline observation respectively.

The Lagos CORS is a single Continuously Operating Reference Station established by the lagos state government under the control and management of the office of the Surveyor General of Lagos State. The Cotonou CORS in Littoral State of Benin Republic is one of the CORS of the International GNSS Service. It has BJCO as its four character ID and stands on a monument 3.9m tall. It utilizes a Trimble NET R5 receiver type and has a GNSS capability of tracking both GPS and GLONASS satellites including other satellite constellations.

6. Data collection and processing

The process of Fast Static survey was done uninterruptedly for a minimum period of 30 minutes for each session. The base station was left static throughout the whole period of data collection while the rover stations were changed after each rover station occupation session.

GNSS survey involving differential correction requires a simultaneous observation of the same satellites by both the rover and base stations for successful baseline processing. This necessitated the continuous operation of the base station throughout the survey.

As stated earlier in the previous section concerning the 24/7 operation of the LAG01 and BJCO CORS, the time and date of observation of all rover stations were noted and the appropriate CORS observation at the CORS control center were downloaded for processing in the GNSS Post Processing software (See Fig 3, 4 & 5).

7. Computation of standard error in relative GNSS positioning

In deriving the mathematical model for the computation of the standard error in relative positioning we would begin the derivative from absolute positioning.

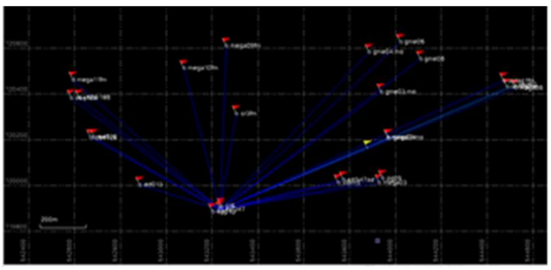

Figure 3: Imagery showing GNSS processed short baselines

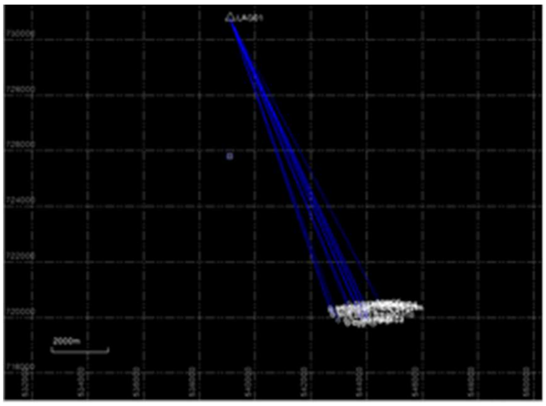

Figure 4: Imagery showing GNSS processed medium baselines

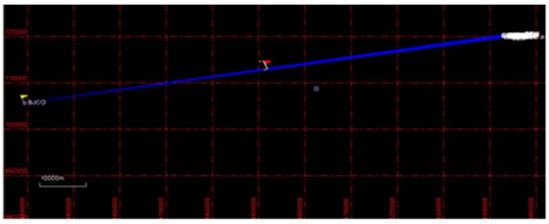

Figure 5: Imagery showing GNSS processed long baselines

Lonchay (2009) presented the following general model for absolute GNSS positioning:

$$p^i_p(t) = D^i_p + T^i_p + I^i_p + M^i_{p,m} + c(\nabla t^i(t_e) - \nabla t_p(t) + \varepsilon^i_{p,m} \quad \ldots(1)$$

T^i_p = Tropospheric Delay

I^i_p = Ionospheric Delay

$M^i_{p,m}$ = Multipath Delay

$\nabla t^i(t_e)$ = Satellite Clock Error

$\nabla t_p(t)$ = Receiver Clock Error

$\varepsilon^i_{p,m}$ = Noise

$$D^i_p = \sqrt{(X^i - X_p)^2 + (Y^i - Y_p)^2 + (Z^i - Z_p)^2} \quad \ldots\ldots(2)$$

The solution is estimated by means of a least squares adjustment.

$$P^i_p(t) = f(X_p, Y_p, Z_p, \nabla t_p) \qquad (3)$$

where the number of unknown parameter is 4(u) and number of redundant observations is ≥ 4(n). From least squares adjustments we therefore have:

$$Ax + W - V = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

$$x = N^{-1}A^TW \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The design matrix A =

$$\begin{bmatrix} \frac{-(X^1 - X_{p,o})}{D^1_{p,o}} & \frac{-(Y^1 - Y_{p,o})}{D^1_{p,o}} & \frac{-(Z^1 - Z_{p,o})}{D^1_{p,o}} \\ \frac{-(X^2 - X_{p,o})}{D^2_{p,o}} & \frac{-(Y^2 - Y_{p,o})}{D^2_{p,o}} & \frac{-(Z^2 - Z_{p,o})}{D^2_{p,o}} \\ \vdots & \vdots & \vdots \\ \frac{-(X^n - X_{p,o})}{D^n_{p,o}} & \frac{-(Y^n - Y_{p,o})}{D^n_{p,o}} & \frac{-(Z^n - Z_{p,o})}{D^n_{p,o}} \end{bmatrix} \ldots\ldots (6)$$

In the design matrix above the number of row of the matrix is equal to the number of redundant observations while the number of column is equal to the number of unknown parameters.

Normal Equation Matrix: $N = A^T A$ ………….. (7)

Cofactor matrix:

$$Q_{\hat{X}} = N^{-1} = \begin{bmatrix} q_{XX} & q_{XY} & q_{XZ} & q_{Xt} \\ q_{YX} & q_{YY} & q_{YZ} & q_{Yt} \\ q_{ZX} & q_{ZY} & q_{ZZ} & q_{Zt} \\ q_{tX} & q_{tY} & q_{tZ} & q_{tt} \end{bmatrix} \ldots\ldots\ldots\ldots (8)$$

It is absolutely important to note that the amplifying effects on the impact of errors from observations to adjusted parameters in determining the standard error in relative positioning is the satellite geometry and the quality indicator of the satellite geometry is the dilution of precision which in the case of relative positioning is the Relative Dilution of Precision (RDOP).

The dilution of precision (DOP) =

$$Q_{\hat{X}} = N^{-1} = \begin{bmatrix} q_{XX} & q_{XY} & q_{XZ} & q_{Xt} \\ q_{YX} & q_{YY} & q_{YZ} & q_{Yt} \\ q_{ZX} & q_{ZY} & q_{ZZ} & q_{Zt} \\ q_{tX} & q_{tY} & q_{tZ} & q_{tt} \end{bmatrix} = (A^T A)^{-1} \ldots\ldots\ldots (9)$$

The general principle of relative positioning also presented by Lonchay (2009) is thus:

$$P^{ij}_{AB}(t) = P^i_{AB} - P^j_{AB} = D^{ij}_{AB} + T^{ij}_{AB} + I^{ij}_{AB} + M^{ij}_{AB,m} + \varepsilon^i_{AB,m} \quad \ldots(10)$$

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T^{i}_{AB} = Tropospheric Delay

I^{i}_{AB} = Ionospheric Delay

$M^{i}_{AB,m}$ = Multipath Delay

$\varepsilon^{i}_{AB,m}$ = Noise

$$D^{ij}_{AB} = \sqrt{(X_A - X_B)^2 + (Y_A - Y_B)^2 + \left(Z_A - Z_B\right)^2} \quad \ldots (11)$$

There is no effect of satellite and receiver clock errors because relative GNSS positioning provides correction for these errors.

$$P^{ij}_{AB}(t) = f(X_B, Y_B, Z_B) \quad \ldots\ldots\ldots\ldots (12)$$

where unknown parameters u = 3 and Redundant Observations n ≥ 3

From least Squares, Relative Dilution of Precision (RDOP) =

$$Q_{\hat{X}} = N^{-1} = (A^T A)^{-1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

The design matrix A =

$$\begin{bmatrix} \frac{-(X_A{}^1 - X_B)}{D^1_{AB}} & \frac{-(Y_A{}^1 - Y_B)}{D^1_{AB}} & \frac{-(Z_A{}^1 - Z_B)}{D^1_{AB}} \\ \frac{-(X_A{}^2 - X_B)}{D^2_{AB}} & \frac{-(Y_A{}^2 - Y_B)}{D^2_{AB}} & \frac{-(Z_A{}^2 - Z_B)}{D^2_{AB}} \\ \vdots & \vdots & \vdots \\ \frac{-(X_A{}^n - X_B)}{D^n_{AB}} & \frac{-(Y_A{}^n - Y_B)}{D^n_{AB}} & \frac{-(Z_A{}^n - Z_B)}{D^n_{AB}} \end{bmatrix} \ldots\ldots\ldots \quad (14)$$

Therefore, the standard error in relative positioning δ_{RPOS} is a function of both the relative dilution of precision and the standard error in range measurements (baseline length) between the base and the rover stations simultaneously acquiring GNSS satellite ephemeris.
Thus:

$$\delta_{RPOS} = (A^T A)^{-1} \delta_r \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

where: δ_{RPOS} = Standard error in relative positioning

$(A^T A)^{-1}$ = Cofactor of adjusted parameter equivalent to Relative Dilution of Precision.

δ_r = Standard error in range or baseline measurements

8. Results and analysis

Tables (1-3) show horizontal and vertical precision the result of GNSS for short, medium and long baselines processing of selected stations. Tables (4-6) show the result of computation of standard error in relative positioning of GNSS for short, medium and long baselines processing.

Table 1: Horizontal and vertical precision of GNSS with short baselines processing of selected stations

Stations	Easting (m)	Northing (m)	Height (m)	Horizontal precision (m)	Vertical precision (m)
Cr 8	543240.659	719908.825	6.247	0.003	0.005
Cblm 3	543750.878	720011.466	7.448	0.002	0.003
XST347az	543773.417	720023.868	8.157	0.003	0.005
Mega 03	543928.957	720011.221	9.848	0.008	0.014
PG 09	543944.031	720030.444	9.814	0.004	0.007
ED 013	542884.766	720001.874	7.855	0.002	0.002
ED 015	542684.951	720210.028	8.715	0.005	0.008
DOS 12S	542670.865	720209.53	8.622	0.011	0.018
DOS 14S	542584.668	720380.971	8.64	0.003	0.005
Ytt 28/186	542621.444	720382.246	8.847	0.009	0.008
Gme 02	543971.894	720208.622	8.076	0.005	0.009
Gme 03	543938.78	720408.336	8.306	0.006	0.012
Cr3 f	543306.243	720312.627	6.515	0.013	0.022
Mega 09	543261.651	720608.475	8.442	0.005	0.009
Mega 10	543077.216	720510.877	8.763	0.007	0.007
Mega 11	542592.889	720460.042	7.663	0.016	0.019
Mega 06	544435.929	720542.61	1.558	0.062	0.132
Unilag 1	544473.004	720456.463	3.962	0.006	0.014
Unilag 2	544488.197	720430.507	3.786	0.06	0.01

Table 2: Horizontal and vertical precision of GNSS with medium baselines processing of selected stations

Stations	Eastings (m)	Northing (m)	Height (m)	Horizontal Precision (m)	Vertical Precision (m)
Cr 8	543330.824	719790.447	6.569	0.007	0.012
Cblm 3	543841.038	719893.072	7.278	0.008	0.011
XST347az	543863.577	719905.477	7.983	0.008	0.014
Mega 03	544019.114	719892.834	9.667	0.007	0.014
PG 09	544034.197	719912.058	9.616	0.023	0.033
ED 013	542974.923	719883.486	7.862	0.008	0.01
ED 015	542775.055	720091.504	8.626	0.013	0.022
DOS 12S	542761.01	720091.148	8.619	0.011	0.023
DOS 14S	544062.056	720090.234	8.084	0.009	0.015
Ytt 28/186	542711.611	720263.866	8.63	0.015	0.031
Gme 02	544062.056	720090.234	8.084	0.011	0.035
Gme 03	544028.938	720289.958	8.31	0.015	0.033
Cr3 f	543396.388	720194.26	6.541	0.007	0.022
Mega 09	543351.809	720490.111	8.459	0.007	0.018
Mega 10	543167.367	720392.512	8.72	0.009	0.022
Mega 11	542683.089	720341.713	7.555	0.013	0.038
Mega 06	544526.948	720424.043	5.857	0.183	0.288
Unilag 1	544563.151	720338.086	3.886	0.011	0.025
Unilag 2	544578.369	720312.131	3.563	0.026	0.076

Table 3: Horizontal and vertical precision of GNSS with long baselines processing of selected stations

Stations	Eastings (m)	Northing (m)	Height (m)	Horizontal Precision (m)	Vertical Precision (m)
Cr 8	543329.842	719789.529	5.911	0.289	0.264
Cblm 3	543840.049	719892.159	6.487	0.242	0.276
XST347az	543862.474	719904.605	5.307	0.373	0.155
Mega 03	544018.315	719892.065	9.048	0.361	0.318
PG 09	544033.377	719911.232	9.315	0.407	0.52
ED 013	542973.912	719882.636	6.826	0.284	0.124
ED 015	542773.782	720090.569	8.358	0.311	0.425
DOS 12S	542759.58	720090.184	7.657	0.39	0.338
DOS 14S	542673.765	720261.667	7.698	0.173	0.096
Ytt 28/186	542710.519	720262.978	8.012	0.346	0.167
Gme 02	544061.121	720089.393	7.359	0.459	0.487
Gme 03	544027.785	720289.102	7.478	0.338	0.227
Cr3 f	543395.358	720193.368	3.864	0.289	0.163
Mega 09	543350.816	720489.253	7.762	0.246	0.241
Mega 10	543166.31	720391.624	8.065	0.199	0.136
Mega 11	542681.875	720340.629	7.192	0.447	0.456
Mega 06	544525.54	720423.329	2.161	0.419	0.496
Unilag 1	544562.093	720337.209	2.971	0.247	0.153
Unilag 2	544577.414	720311.303	2.835	0.207	0.182

Table 4: Results of computation of standard error in relative positioning for GNSS short baselines processing

Station From	Station To	Baseline Length (m)	Standard Error in Range (m)	Max PDOP	Number of GPS Satellite	Number of GLONASS Satellite	Total Satellite Visibility	Standard Error in Relative Positioning (m)
XST347	Cr 8	15.519	0.001	1.335	11	8	19	0.001335
XST347	Cblm 3	528.813	0.001	1.187	11	9	20	0.001187
XST347	XST347az	553.597	0.001	1.394	11	8	19	0.001394
XST347	Mega 03	703.592	0.003	1.692	10	5	15	0.005076
XST347	PG 09	721.848	0.002	1.805	10	6	16	0.00361
XST347	ED 013	366.956	0.001	1.248	11	8	19	0.001248
XST347	ED 015	634.875	0.002	2.06	10	7	17	0.00412
XST347	DOS 12S	646.892	0.004	1.671	7	6	13	0.006684
XST347	DOS 14S	812.967	0.001	1.56	9	4	13	0.00156
XST347	Ytt 28/186	784.61	0.003	1.636	8	4	12	0.004908
XST347	Gme 02	801.069	0.001	1.647	10	5	15	0.001647
XST347	Gme 03	871.543	0.002	1.799	9	4	13	0.003598
XST347	Cr3 f	424.517	0.003	1.638	8	5	13	0.004914
XST347	Mega 09	715.006	0.001	1.788	9	4	13	0.001788
XST347	Mega 10	636.87	0.001	1.692	8	5	13	0.001692
XST347	Mega 11	856.485	0.004	2.379	6	5	11	0.009516
XST347	Mega 06	1364.921	0.023	2.025	9	6	15	0.046575
XST347	Unilag 1	1359.815	0.002	2.687	7	4	11	0.005374
XST347	Unilag 2	1363.241	0.002	1.584	8	4	12	0.003168

Table 5: Results of computation of standard error in relative positioning for GNSS medium baselines processing

Station From	Station To	Baseline Length (m)	Standard error in Range (m)	Max PDOP	Number of GPS Satellite	Number of GLONASS Satellite	Total Satellite Visibility	Standard Error in Relative Positioning (m)
LAG01	Cr 8	11776.372	0.002	1.688	8	8	16	0.003376
LAG01	Cblm 3	11875.899	0.003	1.187	11	9	20	0.003561
LAG01	XST347az	11873.595	0.003	1.394	10	7	17	0.004182
LAG01	Mega 03	11948.632	0.003	1.679	10	5	15	0.005037
LAG01	PG 09	11937.376	0.007	1.805	10	6	16	0.012635
LAG01	ED 013	11564.943	0.003	1.251	10	8	18	0.003753
LAG01	ED 015	11302.298	0.005	2.06	11	7	18	0.0103
LAG01	DOS 12S	11298.051	0.004	1.606	10	7	17	0.006424
LAG01	DOS 14S	11107.859	0.003	1.881	7	6	13	0.005643
LAG01	Ytt 28/186	11118.598	0.006	1.951	8	8	16	0.011706
LAG01	Gme 02	11787.184	0.004	1.539	10	5	15	0.006156
LAG01	Gme 03	11592.228	0.006	1.394	10	5	15	0.008364
LAG01	Cr3 f	11425.169	0.003	1.638	9	6	15	0.004914
LAG01	Mega 09	11134.507	0.003	1.788	10	5	15	0.005364
LAG01	Mega 10	11155.843	0.003	2.638	7	5	12	0.007914
LAG01	Mega 11	11035.673	0.005	1.804	6	5	11	0.00902
LAG01	Mega 06	11694.604	0.039	1.39	6	5	11	0.05421
LAG01	Unilag 1	11787.568	0.004	2.687	9	5	14	0.010748
LAG01	Unilag 2	11817.634	0.01	1.637	7	5	12	0.01637

Table 6: Results of computation of standard error in relative positioning for GNSS long baselines processing

Station From	Station To	Baseline Length (m)	Standard error in Range (m)	Max PDOP	Number of GPS Satellite	Number of GLONASS Satellite	Total Satellite Visibility	Standard Error in Relative Positioning (m)
BJCO	Cr 8	105063.21	0.115	2.131	8	7	15	0.245065
BJCO	Cblm 3	105582.793	0.094	2.002	9	7	16	0.188188
BJCO	XST347az	105606.703	0.148	2.131	6	5	11	0.315388
BJCO	Mega 03	105759.483	0.146	2.295	7	6	13	0.33507
BJCO	PG 09	105776.999	0.164	1.984	9	6	15	0.325376
BJCO	Ed 013	104723.036	0.112	2.027	8	7	15	0.227024
BJCO	Ed 015	104553.255	0.125	1.737	7	7	14	0.217125
BJCO	DOS 12S	104539.131	0.159	2.051	7	6	13	0.326109
BJCO	DOS 14S	104477.99	0.067	2.13	7	5	12	0.14271
BJCO	Ytt 28/186	104514.579	0.14	2.032	8	7	15	0.28448
BJCO	Gme 02	105828.697	0.187	1.752	7	5	12	0.327624
BJCO	Gme 03	105823.145	0.133	2.303	8	5	13	0.306299
BJCO	Cr3 f	105183.314	0.117	1.734	8	6	14	0.202878
BJCO	Mega 09	105180.582	0.098	2.004	8	7	15	0.196392
BJCO	Mega 10	104984.087	0.081	2.628	6	5	11	0.212868
BJCO	Mega 11	104497.112	0.178	3.402	5	4	9	0.605556
BJCO	Mega 06	106334.904	0.169	2.625	6	3	9	0.443625
BJCO	Unilag 1	106359.171	0.099	2.125	7	6	13	0.210375
BJCO	Unilag 2	106370.772	0.082	2.04	8	6	14	0.16728

9. Statistical analysis of results

9.1 One way ANOVA statistical analysis

The GNSS observations of short, medium and long baselines and all subsequent data processing and computations results in population samples standard error in relative positioning for the short, medium and long baselines as shown in tables 1 to 3 above. With these data, the comparative analysis of the results statistically is imperative to achieve the research objectives.

The statistical analyses of the population samples of standard error in relative positioning of the short, medium and long baselines was conducted using One-Way ANOVA (Table 7).

Table 7: Summary results of one way ANOVA

		Sum of Square	df	Mean Square	F
Standard Error in Relative Positioning (m)	Between Groups	0.73	2	0.365	31.33
	Within Groups	0.63	54	0.012	
	Total	1.36	56		

H_O: No differences between the means of the 3 groups
H_A: At least one of the means is not the same as other means ($\alpha = 0.05$)

REJECT H_O at $\alpha = 0.05$
At least one of the means is not the same as other means

The One-Way ANOVA procedure produces a one-way analysis of variance for a quantitative dependent variable by a single factor (independent) variable. Analysis of variance is used to test the hypothesis that several means are equal. This technique is an extension of the two-sample *t* test.

9.2 Scheffe test

Scheffé Test performs simultaneous joint pairwise comparisons for all possible pairwise combinations of means. Uses the F sampling distribution. Can be used to examine all possible linear combinations of group means, not just pairwise comparisons.

Scheffé test is a post Hoc test applied after determining that differences exist among the means in One-Way ANOVA, Scheffé tests determines which means differ (SPSS, 2007). Post Hoc tests are also regarded as orthogonal contrast employed in knowing which means are significantly different and which ones are not.

10. Discussions

All foregoing results were obtained from observations, processing and analysis of short, medium and long GNSS baselines. These are presented in both, tabular and graphical forms for easy interpretations.

Tables 1, 2 and 3 show the results of the spatial coordinates as well as resulting horizontal and vertical precision for the short, medium and long baselines GNSS processing of the observed stations consecutively. The horizontal and vertical precision is a measure of accuracy in determining the X, Y, Z position of the observed stations.

Table 8: Results of Scheffé test

	SAMPLE TO SAMPLE		F	P(>F)	COMMENTS @ (α = 0.05)
Standard Error in Relative Positioning (m)	Short Baseline	Medium base-line	0.02	0.98	
	Short Baseline	Long Base-line	86.9	0	Short Baseline Mean < Long Baseline Mean
	Medium Baseline	Long Base-line	84.2	0	Medium Baseline Mean < Long Baseline Mean

Tables 4, 5 and 6 show the standard error in relative positioning of the selected stations with respect to short, medium and long baseline observation scenarios respectively. It should be noted that the standard error in relative positioning is a function of the satellite geometry which is defined by the satellite visibility and the PDOP. The number of visible satellites both for GLONASS and GPS constellations as well as resulting PDOP for each station, can be observed from the same tables. The higher the PDOP values the poorer the satellite geometry compared to low PDOP values. This consequently affects the standard error in relative positioning and the variations are clearly seen in the tables with respect to individual stations for all the three baseline observation scenarios.

The graphical illustrations in figure 6 shows the variation of standard error in relative positioning for the short, medium and long baselines. A comparative analysis of the standard error in relative positioning with respect to the three baseline scenarios can be graphically inferred from the chart. In the graphical illustration, the long baseline observation have the highest standard error in relative positioning while the short baseline observations have the least standard error in relative positioning. However, *Cs3f* station have equal standard error in relative positioning in the short and medium baseline processing while *Mega11* station have lower standard error in relative positioning in the medium baseline observation compared to the short baseline.

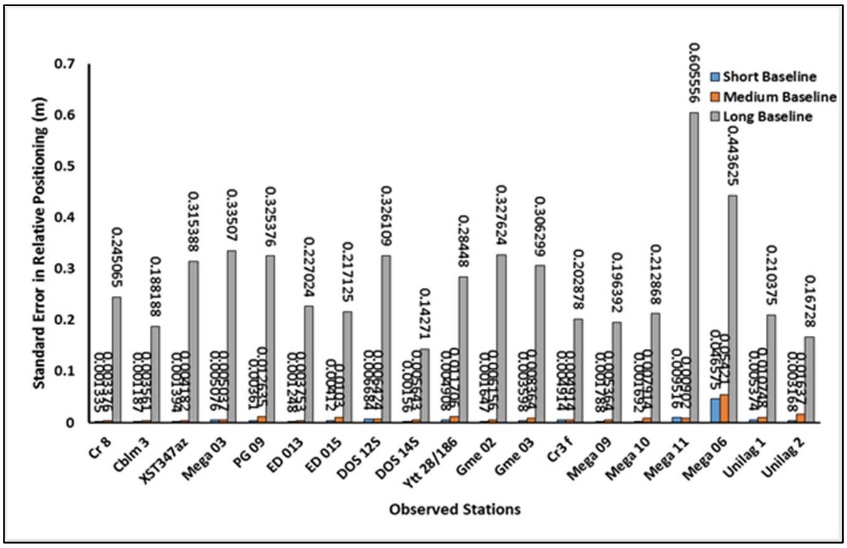

Figure 6: Graphical comparison of standard error in relative positioning at the short, medium and long baseline observations

The foregoing discussions indicates variations in the results of the short, medium and long baseline processing scenario. These analyses would not be complete without an overall statistical analysis between the results of the three baseline processing scenarios. Table 7 shows the results of one way ANOVA statistical test performed on the standard error in relative positioning results for the short, medium and long baselines. The test was conducted at 95% confidence interval; that is, 0.05 significant level. The null hypothesis states that there is no significant difference between the results of the three baseline processing scenarios while the alternative states otherwise. The end results of the test indicate that there exist significant difference in at least one of the data set compared to another. This existing difference requires a further testing to ascertain where the difference lies. Thus further Post-Hoc test was conducted using Scheffé statistical testing.

The results of the Scheffé test indicates that there is no significant difference in the results of the short and medium baseline but there is a significant difference in the results of the short and long baseline and the results of the medium and long baseline. The Scheffé test was also conducted at 0.05 significant level. The summary of the Scheffé result is presented in table 8.

11. Conclusion

The present study has shown that GNSS baseline processing is dependent on the baseline length. The longer the baseline length the lower the attainable precision. The standard error in relative GNSS positioning of all the observed stations varies as the baseline length changes. The variation is not only dependent on static conventional base station but also on CORS. The statistical tests indicated the progressive error propagation in positional accuracy as the medium and short baseline results show no significant difference but they both statistically differ from the long baselines. The research has thus, justified the importance of understanding the concept of baseline processing in GNSS positioning as it has a significasnt impact on the achievable positional accuracy, both for conventional base stations and CORS.

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