Adjustment of DGPS data using artificial intelligence and classical least square techniques

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Abstract: To improve on the accuracy of survey works, there is the need for proper adjustment of survey data. Adjustments and computations of survey field data has played a vital role in mathematical geodesy, it has been applied for the study of magnitude of errors and the determination of tolerance levels. Several studies have been carried out over the years in adjusting survey field data through the application of classical least squares techniques and other methods. With current increase in usage of GPS for most ground truth survey works, the need to adjust field data after post processing have not been taken seriously resulting in suspicion in the accuracy of final output of GPS surveyed data. This study evaluates and test alternative techniques in adjusting Differential Global Positioning System (DGPS) survey field data. Hence, the objective of this study was to explore the efficiency and performance of two artificial intelligence techniques namely, Back propagation Artificial Neural Network (BPANN), and Multivariate Adaptive Regression Spline (MARS) as a realistic alternative technique in adjusting DGPS survey field data. The study also compares the performance of BPANN and MARS models to two classical techniques namely: Ordinary Least Square (OLS) and Total Least Square (TLS). The statistical findings revealed that, BPANN, OLS, and TLS offered satisfactory results in adjusting the DGPS field data. Also, the MARS model compares to BPANN model showed better stability and more accurate results in adjusting the DGPS field data. In terms of their two-dimensional mean horizontal error, the BPANN model attained 0.0654 m while MARS model achieved 0.0296 m as compared to OLS and TLS model which archived 3.3975E -06 m and 1.0027E-09 m respectively. This present study, can conclude that BPANN and MARS provides a promising alternative in the adjustment of DGPS survey field data for Cadastral and Topographic surveys.

Keywords: Adjustment, DGPS, MARS, BPANN, Classical Least Squares

1 Introduction

Survey field measurements since generation are usually compromised with errors in field observations and needs to be adjusted using mathematical models (Okwuashi, 2014). There are two types of survey measurements techniques namely, the direct technique and the indirect technique. The direct techniques are the actual collection of field data. Errors may be incorporated due to personal, the type of instrument used, and the type of survey techniques applied. Indirect techniques are the alternate techniques of achieving field data. In this technique, errors involve in the direct method may propagate into the indirect techniques (Ghilani and Wolf, 2014). Hence, the field data needs to be adjusted to minimize the errors using both survey techniques.

Adjustments and computations studies have become obligatory in the field of mathematical geodesy, to study the magnitude of the errors whether these errors are acceptable and within tolerance limits (Ghilani and Wolf, 2012). In the past centuries, the least squares mathematical regression models (LS) (Gauss, 1823) adjustment techniques was developed and have been applied in many fields. LS is the classical technique for adjusting surveying measurements (Okwuashi and Asuquo, 2012). The LS technique minimizes the sum of the squares of differences between the observation and the estimate (Bezrucka, 2011). Various techniques utilized in the recent and past decades include, Kalman Filter (KF) (Kalman, 1960), Least Squares Collocation (LSC) (Moritz, 1978), and Total Least Square (TLS) (Golub and Van Loan, 1980; Akyilmaz, 2007; Annan *et al*., 2016a, 2016b). In this study, two classical techniques namely the ordinary least square (OLS) and total least square were adopted to assess the performance of two artificial intelligence techniques namely Multivariate Adaptive Regression Splines (MARS) and Backpropagation Neural Networks (BPANN) as an alternative technique in adjusting field data due to some defects with the classical methods.

The Ordinary Least Square (OLS) have been the conventional techniques for adjusting surveying networks (Okwuashi and Eyoh, 2012). OLS only considers the observations equations to be stochastic (Acar *et al*., 2006) and adjust only the errors in the observation matrix to make the square of the sum of residuals minimum. Several researchers (Annan *et al*., 2016a; Ziggah *et al*., 2013) in the field of geoscientific studies have applied OLS to solve many scientific problems. The Total Least Square TLS) is a data modelling technique which can be used for many types of statistical analysis such as regression or classification. In the regression technique, both dependent and independent variables are measured with errors. Thereby, the TLS approach in statistics is sometimes called an errors-in-variables (EIV) modelling. Moreover, this type of regression is usually known as an orthogonal regression (Golub and Van Loan, 1989). The total least squares (TLS) was invented to resolve the working efficiency of the OLS (Annan *et al*., 2016a). The TLS can adjust the errors in both the observation matrix and design matrix (Acar *et al.*, 2006) to yield a better estimate. Researchers such as (Acar *et al.*, 2006; Annan *et al.*, 2016a, 2016b; Okwuashi and Eyoh, 2012) have applied TLS to solve many scientific problems and they concluded that, the TLS working efficiency is encouraging. For large sample properties of the TLS estimator, that is a strong and weak consistency, and an asymptote distribution. The

standard procedure for solving TLS problem involves the singular value decomposition (SVD) of the extended data matrix (Lemmerling *et al*., 1996). However, the SVD does not preserve the structure of the extended data matrix. This implies that the TLS approach will not yield the statistically optimal parameter vector in the frequently occurring case where the extended data matrix is structured (Golub and Van Loan, 1989). In view of this, MARS and BPANN were adopted to evaluate its efficiency and performance as an alternative adjustment technique for adjusting DGPS field measurement data.
In the recent times, artificial neural network (ANN) has been widely adopted and applied to different areas of mathematical geodesy. Its suitability as an alternative technique to the classical methods of solving most geodetic problems has been duly investigated (Ziggah *et al*., 2016a). Some of the problems solved in mathematical geodesy include GPS height conversion (Fu and Liu, 2014; Liu *et al*., 2011), geodetic deformation modelling (Bao *et al*., 2011; Du *et al*., 2014), earth orientation parameter determination (Liao *et al*., 2012). ANN are being criticized for its long training process in achieving the optimal network's topology, and it is not easy to identify the relative importance of potential input variables, and certain interpretive difficulties (Lee and Chen, 2005; Samui, 2013). For this reason, the MARS model was also adopted. BPANN and MARS are both machine learning techniques.

Multivariate Adaptive Regression Splines (MARS) is an adaptive modelling process invented by Friedman (1991) used for non-linear relationships. In addition, MARS divides the predictor variables into piece-wise linear segments to describe non-linear relationships between the predictor and the dependent variable (Leathwick *et al*., 2005; Samui, 2013). There is limited availability of literature of MARS in survey field adjustment studies, but many studies have successfully applied MARS for solving different problems in engineering. Some of the areas of applications include estimating energy demand (Alreja et. al. 2015), slope stability analysis (Samui, 2013; Lall *et al*., 1996).

The existing knowledge and publications have not fully addressed the issue of applying alternative techniques in the adjustment of DGPS field data. In addition, upon careful review of existing studies, the authors realized that the utilization of the BPANN and MARS techniques have not been applied as a practical alternative technology to the existing approaches. This present study for the first time explored the utilization of the BPANN and MARS in the adjustment of DGPS data. To achieve the aim of this present study, the ANN and MARS methods were applied. This study also highlights the comparison between BPANN and MARS to two classical techniques namely the ordinary least square (OLS) and total least square (TLS). Each model was assesses based on statistical performance indicators such as mean horizontal error (MHE), mean square error (MSE), and standard deviation (SD). The statistical findings of these two models (BPANN and MARS) will reveal the working efficiency and performance of the models for adjustment of the DGPS data. Hence this study will serve as an added contribution to existing knowledge of ANN and MARS in mathematical geodesy.

2 Resources and methods used

The study area (Figure 1) is in the Southwest of Ghana with geographical coordinates between longitudes: 2 º 05 ′ 00 ″ W, and 2 º 45 ′ 00 ″ W; and latitude 4 º 55 ′ 00 ″ N, and 5 º 30 ′ 00 ″ N. The type of coordinate system used in the study area is the Ghana projected grid derived from the Transverse Mercator 1 º NW and the (WGS84) (UTM Zone 30N).

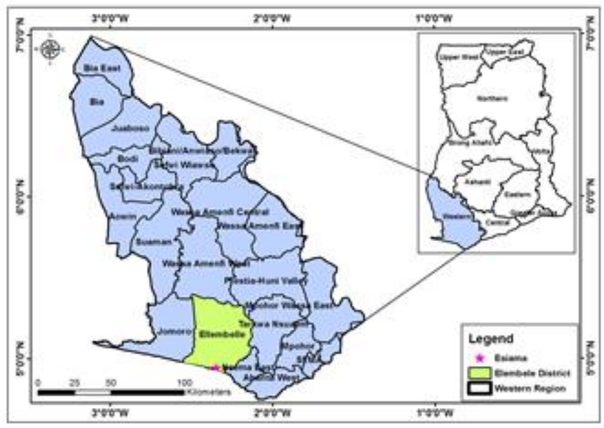

Figure 1: Map of the Study Area

In this study, a total of 53 DGPS data collected by field measurements around Esiama in the Elembelle District of the Western Region, Ghana-West Africa, were used in the model's formulation. It is well acknowledged that, one of the contributing factors affecting the estimation accuracy of models is related to the quality of datasets used in model-building (Dreiseitl and Ohno-Machado, 2002; Ismail *et al*., 2012). Therefore, to ensure that the obtained field data from the GPS receivers are reliable, several factors such as checking of overhead obstruction, observation period, observation principles and techniques as suggested by many researchers (Yakubu and Kumi-Boateng, 2011; Ziggah *et al*., 2016b) were performed on the field. In addition, all potential issues relating to DGPS survey work were also considered. Table 1 shows a sample of the data used to embark on this study. The differential (relative) was adopted in the collection of data due to its ability to adjust and compensate for errors in the baselines measurements.

2.1 Methods

2.1.1 Backpropagation Artificial Neural Network (BPANN)

BPANN consist of three layers namely, the input layer, hidden layer and output layer. In BPANN model formulation, the dataset must be normalized. Before using the dataset for training, it was ensured that the dataset are free from systematic and gross errors, random errors are the only error in the data which follows a normal distribution. The data to be used for the BPANN training and its model formulation are expressed in different units with different physical meanings. Therefore, to ensure constant variation in the BPANN model, datasets are frequently normalized to a certain interval such as [-1, 1], [0, 1] or other scaled criteria (Ziggah *et al.*, 2016b). The selected input and output variables were normalized between the intervals [−1, 1] according to Equation 1 denoted as (Muller and Hemond, 2013):

$$y_i = y_{\min} + \frac{(y_{\max} - y_{\min}) \times (x_i - x_{\min})}{(x_{\max} - x_{\min})} \qquad (1)$$

where y_i represents the normalized data, x_i is the measured coordinates, while $x_{\min}$ and $x_{\max}$ represent the minimum and maximum values of the measured coordinates with $y_{\max}$ and $y_{\min}$ values set at 1 and -1, respectively.

Table 1: Sample of data used for the study (Units in meters)

Northings (m)	Eastings (m)
29516.8527	125559.5038
29612.3465	125168.1490
29695.5483	124868.5474
29766.1450	124750.3910
29774.2700	124741.4477
29813.5507	124768.5206
29786.7162	124619.4892
29847.7689	124426.1192
30133.1594	123937.8123
30174.3410	123719.6955

To find the optimum weight combination, the network was trained up using Bayesian Regularization learning algorithm. The datasets were divided into training (70 %) and testing (30 %). The input variables were the eastings and northings denoted as (E_{input}, N_{input}) and the output were the eastings and northings denoted as (E_{output}) and (N_{output}) respectively. The objective of training is to find the set of weights between the neurons that determine the global minimum of error function. The main function of the testing set is to evaluate the generalization ability of a trained network. Training is stopped when the error of the testing set starts to increase (Chakraborty and Goswani, 2017). The coefficient of correlation (R) and the mean square error (MSE) are the main performance criteria indices that are often used to evaluate the prediction performance of ANN models. The MSE is represented by Equation (2) and R is represented by Equation (3) respectively as:

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(\alpha_i - \beta)_i^2 \qquad (2)$$

where α_i and β are the measured and predicted plane displacements from the BPNN model.

$$R = \frac{\sum_{i=1}^{N}\left(Z_{jai} - \overline{Z}_{ja}\right)\left(Z_{jpi} - \overline{Z}_{jp}\right)}{\sqrt{\sum_{i=1}^{N}\left(Z_{jai} - \overline{Z}_{ja}\right)}\sqrt{\left(\sum_{i=1}^{N} Z_{jpi} - \overline{Z}_{jp}\right)}} \qquad (3)$$

where Z_{jai} and Z_{jpi} are the actual and predicted Z_j values, respectively. $\overline{Z}_{ja}$ and $\overline{Z}_{jp}$ are the mean of actual and predicted Z_j values corresponding to N patterns. For a good model, the value of R should be close to one (Samui, 2013).

2.2.2 Multivariate Adaptive Regression Splines (MARS)

The MARS model is nonparametric (Friedman, 1991) and it works by dividing the variables into regions, producing each region a linear regression equation (Leathwick *et al.*, 2005). The general formula for the MARS model adopted in this study is given by Equation 4 as denoted by (Samui and Kurup, 2012):

$$y(i) = f(x) = a_0 + \sum_{n=1}^{N}\alpha_n\beta_n(x) \qquad (4)$$

Where, $y(i)$ is the dependent variable (measured data) predicted by the function $f(x)$, a_0 is a constant, and N is the number of terms, each of them formed by a coefficient α_n and $\beta_n(x)$ is an individual basis functions or a product of two or more basis functions. The MARS model was developed in two steps. In the first step (the forward algorithm), basis functions are presented to define Equation 4. Many basis functions are added in Equation 4 to get a better estimate of the dependent value (Samui and Kim, 2012). The developed MARS may experience overfitting due to the large number of basis functions used (Friedman, 1991). To mitigate this problem, the second step that is the backward algorithm prevents overfitting by removing redundant basis functions from Equation 4. The MARS model adopts Generalized Cross-Validation (GCV) to delete the redundant basis functions (Samui and Kothari, 2012). The expression of GCV is given by Equation 5 written as (Craven and Wahba, 1979):

$$GCV = \frac{\frac{1}{N}\sum_{i=1}^{N}\left[y_i - \dot{f}(x_i)\right]^2}{\left[1 - \frac{C(H)}{N}\right]^2} \qquad (5)$$

Where N is the number of data and $C(H)$ is a complexity penalty that increases with the number of basis function (BFs) in the model and which is defined as denoted by Equation 6:

$$C(H) = (h+1) + dH \qquad (6)$$

Where d is a penalty for each BFs included into the model and H is the number of basis functions in Equation 4 (Friedman, 1991; Samui and Kothari, 2012). In this present study, the salford predictive model software (SPM) was adopted to train the MARS model. This is because, the SPM software is designed to be highly accurate, ultra-fast analytics, and data mining platform for creating predictive, descriptive and analytical models from databases of any size (Anon., 2018).

2.1.3 Ordinary Least Square (OLS) and Total Least Square (TLS)

OLS is used to solve a system of over determined equations as given by Equation 7 as (Miller, 2006):

$$AX = L + V_L \quad (7)$$

The solution by OLS is given by Equation 8:

$$X = inv(A' * PA) * (A' * PL) \quad (8)$$

The error vector V_L associated with the OLS is given by Equation 9 as (Schaffrin, 2006):

$$V_L = AX - L \quad (9)$$

TLS is a method of treating an over determined system of linear equations by solving for the unknown parameters, $\hat{X}$ in Equation (10) (Golub and Van Loan, 1980) through the form:

$$L + V_L = (A + V_A)\hat{X}, \quad rank(A) = m < n, \quad (10)$$

Where V_L and V_A is the vector of errors in the observation and the data matrix. Both V_L and V_A are assumed to have independent and identical distributed rows with zero mean and equal variance (Felus and Schaffrin, 2005; Akyilmaz; 2007). The TLS method is an iterative algorithm that minimizes the errors through a minimizing matrix $\left[\hat{A}, \hat{L}\right]$. The iteration continues until any $\hat{X}$ that satisfies $\hat{A}\hat{X} = \hat{L}$ becomes the TLS solution (Golub and Van Loan; 1980; Yanmin *et al.*, 2011). The singular value decomposition (SVD) of the matrix $[A, L]$ was used in solving the TLS problem. SVD is used to present $[A, L]$ through Equation (11) as denoted by:

$$[A, L] = USV^T \quad (11)$$

Where $U = [U_1, U_2], U_1 = [U_1, \cdots, U_m]$, $U_2 = [U_{m+1}, \cdots, U_n]$, $U^T U = In$ and $U_i \in R^n$. $V = [V_1, \cdots, V_m, V_{m+1}]$, $V^T V = I_{m+1}$ and $V_i \in R^{m+1}$.

$S = diag(\delta_1, \cdots, \delta_m, \delta_{m+1})$, $S \in R^{n(m+1)}$. Through the SVD, the solution for the TLS problem is finally given by Equation 12 as:

$$\left[\hat{X}^T, -1\right]^T = \frac{-1}{V_{m+1}, m+1} * V_{m+1} \quad (12)$$

If $V_{m+1}, m+1 \neq 0$, then $\hat{L} = \hat{A}\hat{X} = -1/(V_{m+1}, m+1)\hat{A}[V_1, m+1, \cdots, V_m, m+1]^T$ which belongs to the column space of $\hat{A}$, so $\hat{X}$ solves the basic TLS problem. The corresponding TLS correction is expressed by Equation (13) by:

$$\left[\Delta\hat{A}, \Delta\hat{L}\right] = [A, L] - \left[\hat{A} - \hat{L}\right] \quad (13)$$

2.2 Models Performance Assessment

To compare the results obtained from the BPANN, MARS, OLS and TLS model, the residuals computed between the measured coordinates and the adjusted coordinates were used. The statistic indicators used include the Mean Squared Error (MSE), Root Mean Square Error (RMSE), Horizontal Position Error (HE), Mean Horizontal Position Error (MHE) and Standard Deviation (SD). The mathematical expression for the various performance indices are given by Equations 14 to 17 respectively.

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(O_i - P_i)^2 \quad (14)$$

$$HE = \sqrt{(E_2 - E_1)^2 + (N_2 - N_1)^2} \quad (15)$$

$$MHE = \frac{1}{n}\sum_{i=1}^{n} HE_i \quad (16)$$

$$SD = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(e - \bar{e})^2} \quad (17)$$

With reference to Equations (14) to (17), n is the total number of points, O and P are the measured coordinates and adjusted coordinates produced by the various methods applied. *e* represents the residuals between the existing and transformed projected grid coordinates and $\bar{e}$ is the mean value of the residuals.

3 Results and discussions

The Tansig and Purelin functions were used for both the hidden and output layer respectively. The optimal model in adjusting the DGPS data by the BPANN model was [2, 1, 1] for the eastings and [2, 1, 1] for the northings. Thus, two inputs variables, one hidden neuron and one output variable respectively. In the MARS model formulations, 20 basis functions were in the training and testing for adjusting the DGPS data. In terms of the eastings, 6 basis functions were used in the final model formulation. This implies that, 14 basis functions were removed during the backward training due to overfitting. For the northings, 4 basis functions were used in the final model formulation. Table 2 shows the model performance for BPANN and MARS model and figure 2 represent the horizontal displacement graph by the BPANN and MARS model. It can be observed that the MARS model outperforms the BPANN model in adjusting the DGPS field data with better accuracy. The capabilities of MARS in achieving a better result in this study as compare to BPANN may be due to its less time in training the dataset. The correlation coefficient (R) which shows how close the estimated values are to the measured values were approximately one. This implies there is a stronger correlation between the independent variables (input data) and dependent variables (output data). Hence, both models can be successfully used to adjust the DGPS field data.

The basis functions used for the final model formulations by the MARS model is tabulated in table 3. Equation 18 and Equation 19 is the optimal model equation in adjusting the eastings and northings by the MARS model respectively.

Table 2: BPANN and MARS Model (Units in meters)

MODEL	BPANN		MARS	
PCI	MSE	R	MSE	R
Training Easting	0.00290	0.99999	0.00023	0.99999
Testing Easting	0.00590	0.99998	0.00886	0.99990
Training Northing	0.00380	0.99999	4.4169×10^{-6}	0.99999
Testing Northing	1.12620	0.99999	5.87177×10^{-6}	0.99999

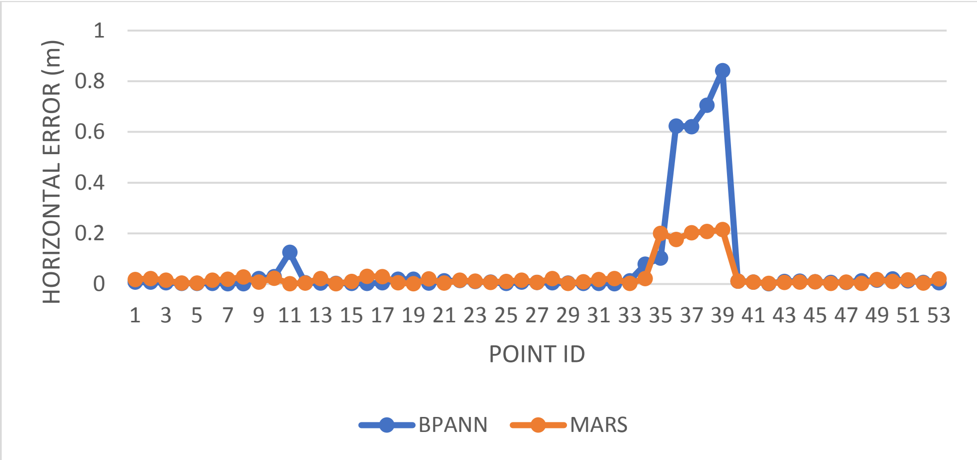

Figure 2: Horizontal Shift Graph of MARS and BPANN Models

Table 3: Basis Functions used by the MARS Model

Eastings	Northings
$BF1 = \max(0, E - 403194);$	$BF1 = \max(0, N - 97656.9);$
$BF6 = \max(0, E - 417262);$	$BF2 = \max(0, 97656.9 - N);$
$BF8 = \max(0, E - 419430);$	$BF3 = \max(0, N - 92505.5);$
$BF10 = \max(0, N - 89823.3);$	$BF6 = \max(0, E - 410677) * BF3;$
$BF11 = \max(0, N - 92505.5);$	
$BF15 = \max(0, N - 97226.4);$	

$$E(i) = 403194 + 0.999992 \times BF1 + 4.39746e - 006 \times BF6 + 3.59577e - 006 \times BF8 + 2.01793e - 006 \times BF10 - 1.56173e - 005 \times BF11 - 1.78743e - 006 \times BF15 \quad (18)$$

$$N(i) = 97656.9 + 1 \times BF1 - 1 \times BF2 + 4.1672e - 012 \times BF3 - 1.00e - 004 \times BF6 \quad (19)$$

The horizontal displacement graphs of the two classical techniques is represented by figure 3. From figure 3, it can be observed that the TLS outperforms the OLS technique due to its capabilities to model out both errors in the observation and design matrix. The OLS model only considers errors in the observation matrix and the remaining ones in the design matrix are considered as uncertainties. Table 4 shows the horizontal displacement results by all the models. From table 4, it can be observed that the results achieved by TLS model was very encouraging as compared to the MARS, BPANN and OLS model which perform better but with less accuracy. Upon carefully analyzing the results, it was realized that the MARS model can be used as a realistic alternative technique to the classical least squares (OLS and TLS) in adjusting some survey field data due to its performance in achieving satisfactory result in this present study. The BPANN which is also a machine learning technique achieved a better result but with lesser accuracy. This implies that, the BPANN may not be an alternative adjustment technique to the classical techniques for the study area. The developed MARS equations given by Equation 15 and Equation 16 can be used for adjusting DGPS data for the study as an alternative technique to the classical methods

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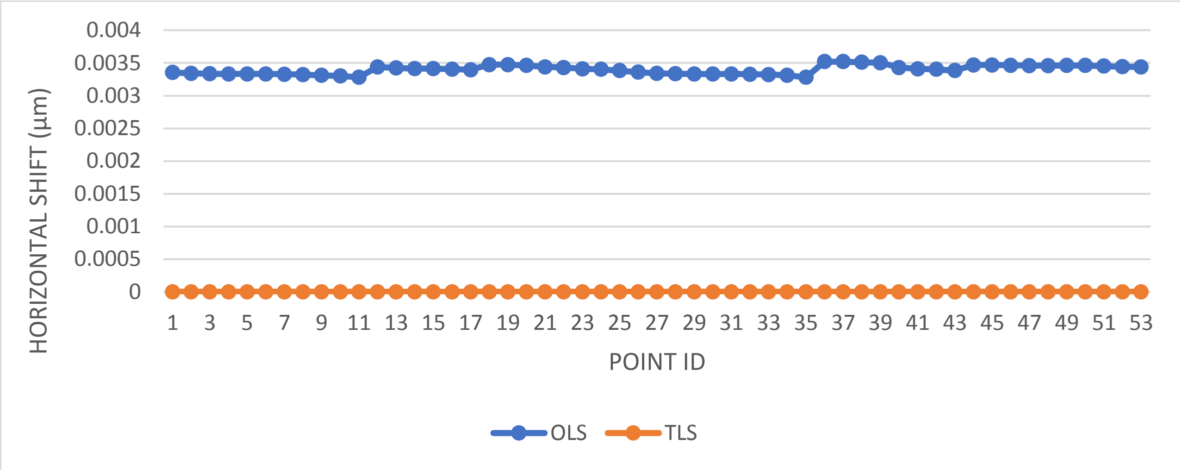

Figure 3: Horizontal displacement graph by the OLS and TLS model

Table 4: Horizontal displacement results by the models (Units in metres)

PCI	MHE	MSE	SD
MARS	0.0296	0.0562	0.0039
ANN	0.0654	0.0380	0.0344
OLS	3.3975E-06	1.1547E-11	4.4329E-15
TLS	1.0027E-09	1.0060E-18	6.0484E-22

The mean horizontal error (MHE) graph by all the models is represented by figure 4 below. From the graph below, TLS, OLS, and MARS can use as an adjustment technique for the study area due to their minimum horizontal shift achieved in this study.

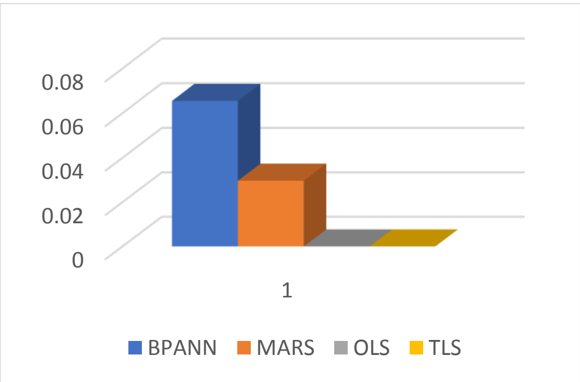

Figure 4: Mean Square Error Graph of the Models

4 Conclusions and recommendations

Adjustments and computations studies of survey field data have become obligatory and common practice in mathematical geodesy to assess the magnitude of errors and to evaluate whether the errors are within acceptable tolerance. However, the existing knowledge has focus more on classical techniques in adjustment of survey data. In the area of DGPS data collection, little knowledge exist in terms of the use of artificial intelligence in adjusting field data post processed.

Artificial intelligence has been applied in this study to adjust DGPS survey field data after the use of the classical least squares techniques. This study is the first time of utilizing artificial intelligence (BPANN and MARS) theoretically and practically on DGPS field data post processed. Machine learning techniques of BPANN and MARS have been presented in this study.

The statistical findings revealed that the BPANN, MARS, OLS and TLS offered satisfactory results in the adjustment of DGPS post processed data. However, MARS model showed superior stability and more accurate adjustment of the DGPS data compared to BPANN model. It can be therefore be proposed that the MARS model can be used as a realistic alternative technique considering certain degree of accuracies to the classical OLS and TLS models within the study area in adjusting DGPS post processed data. Based on the analysis of results achieved, the Eastings and Northings of the DGPS data can be incorporated into the MARS models to give a better estimate as the actual positioning of the ground control point. Therefore, this study does not only have a localized significance but will also open more scientific discourse into the applications of MARS techniques in solving some of the problems in mathematical geodesy within the geoscientific community.

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