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Comparison of Global Geoid Models Against the GPS/Levelling-Derived Geoid Heights in Tanzania

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Abstract: This paper compares the geoid heights from the Global Models – EGM2008, and EGM1996 against the GPS/Levelling-derived geoid heights in Tanzania. For the sake of comparison, the existing Preliminary African Geoid Model (AGP03) and the Tanzania Geoid Model (TZG13) are also tested against the GPS/levelling derived geoid heights at 13 benchmarks selected within the Tanzania Primary Levelling Network (TPLN). The comparisons of geoid heights obtained from these geoid models against the GPS/levelling geoid heights have been performed in absolute sense. Due to the fact that the ellipsoidal heights (h) obtained from the GPS do not provide the actual positions of points on the geoid, the orthometric heights (H) are needed. Broadly speaking, the orthometric heights are obtained through traditional sprit levelling which is a labour intensive work. In order to convert the ellipsoidal height (h) determined from GPS applications to orthometric height, the Geoid heights are needed. The spatial positions of these benchmarks have been recently determined at cm-level accuracy (with respect to ITRF2005) through a GPS campaign. The statistics of the differences between GPS/levelling-derived geoid heights (N_{GPS}) and the corresponding geoid heights obtained from the available three geoid models (N_{model}) suggests that, AGP03 model is the most suitable at this moment. The Root Mean Square (RMS) fit of the AGP03 geoid model against the GPS/levelling data is 53.8 cm, which is a 2 times better fit compared to the Global Geopotential Models (EGM08 and EGM96) in the area of interest. On the other hand, the RMS of the height differences between the TZG13 and the GPS/leveling derived heights was 74.7cm. The study suggests that AGP03 geoid model is closer to the GPS/levelling geoid observations in comparison to EGM08 model in Tanzania.

Key words: Geoid Models, GPS/Levelling, Geoid, TPLN, Tanzania

1. Introduction

The demand for height information from the satellite users based positioning techniques, mostly Global Positioning System (GPS), has increased interest on determination and use of precise geoid models. The knowledge of the local geoid surface allows the transformation of ellipsoidal heights to physically meaningful orthometric heights which are essential in most of the geodetic applications. Thus, GPS measurements in combination with a precise geoid model are preferred in obtaining orthometric heights instead of spirit levelling measurements, which is labour-intensive and costly (Sideris et al., 1992). Normally, for the purpose of GPS/levelling, in the absence of a publicly available geoid model, it is beneficial to select a Global Geopotential Model (GGM) which is a best fit to the local gravity field as the basis for local or regional geoid model (Kiamehr and Sjöberg, 2005). The Global models have a long and important history in the geodetic community, specifically as a tool for computing geoid heights.

On the other hand, many applications in geodesy, geophysics and engineering require physically defined heights related to the earth's gravity field (orthometric or normal heights), typically produced by spirit leveling. Therefore, for the conversion and combination of these fundamentally different height systems, the geoid must be known with accuracy comparable to the accuracy of GPS and leveling. Because more Global Geoid Models (GGMs) have now been released into the public domain, particularly those including data from the CHAMP and GRACE satellites dedicated gravimetry missions, and new gravity-field-related datasets, it is important to make validations in order to select the most appropriate geoid model. The development of the Earth Gravitational Model 2008 (EGM08) by the US National Geospatial-Intelligence Agency (NGA), (Pavlis et al., 2008) and other recent models revealed a major achievement in global gravity field mapping. The model is complete up to degree and order 2159 and contains additional spherical harmonic coefficients extending up to degree 2190 and order 2159, respectively. The EGM08 can provide long and medium wavelength information of the earth's gravity field to a higher resolution of wavelengths equivalent to 10' of arc. However, there are various recent models developed including EIGEN-6C2, EIGEN6C4, EIGEN6S4, GGM05C, GECO, GOCO05c etc. which have high precision and spatial resolutions of gravity data (Pal et al., 2016). This paper compares EGM2008 and local geoid models with the GPS/levelling observations in Tanzania.

Since the release of the global geoid models including the EGM2018 to the earth science community, there has been a strong interest among the geodesists to quantify its actual accuracy with different validation techniques and external datasets independently of the estimation and error estimation procedures that were used for its development (Yilmaz, and Karaali, 2010). Yilmaz and Karaali attempted over Turkey Landmass/Ocean and observed that the global geoid that best fits the GPS/leveling derived geoid heights was EGM08. As currently in Tanzania there is no comprehensive national geoid model which has been released for the public use, several attempts have been done to develop and validate models that best fit in the Tanzania region. Mayunga

(2016, p. 268, cited in Silyvester, 2013) indicates the developed model which was used to compute point values of a gravimetric geoid using short wavelength which later on were compared with GPS/leveling derived geoid heights. The differences obtained and the biases between the geometric and gravimetric geoid models were recorded. Ulotu (2009) developed the gravity database using sparse gravity data with varying density, distribution and quality. Assessment of this model was done by using KHT method and Least Squares Modification of Stokes to compute geoid of Tanzania, and the accuracy obtained was 29.7 cm.

In selection of suitable geoid model for application purposes, Kiamehr & Sjöberg (2005) cautioned and verified that published error estimates for the geoid models, particularly the global models should not be used directly to judge the most suitable Global Geopotential Model (GGM) for a certain regional/local geoid model representation, but rather as performance indicator. The reason is that such performance evaluations sometimes tend to be too pessimistic and global statistics are not necessarily true representatives in a particular region.

Therefore, the user of a GGM should perform his own accuracy and precision verifications, such as comparing the GGM-derived gravity field quantities with local data (Lambeck & Coleman, 1983). The global geoid model data have never been validated in Tanzania for public purposes. In this context, there is a need of validating the global models so that, it can be used in local areas to serve the communities. The validation process should be done by comparing geoid heights obtained from the global models against the GPS/levelling derived geoid heights. However, it is worth mentioning that though many researchers have reviewed the need of National Geoid Model in Tanzania, such as Ulotu (2009) and Mayunga (2016), the validation context has always remained a research area of interest due to lack unified geodetic network.

This paper checks the compatibility of the EGM08, EGM96, AGP03 and the TZG13 geoid models against the existing GPS/levelling - derived geoid in Tanzania by using the weighted mean approach. The purpose of this paper is to present the global geoid (EGM08 and EGM96) and local geoid (AGP03 and TZG13) heights validation on a certain part of the Tanzania Primary levelling Network (TPLN) by GPS/leveling.

2. Geoid, Ellipsoid and Orthometric Heights: A theoretical framework

2.1. Geoidheight

To understand what geoid heights mean, it is imperative to know what does geoid entails. The geoid is an equipotential surface of the earth's gravity field which, a least square sense coincides with mean sea surface in the open ocean. It is a best fit mean sea level surface. The geoid serves as a reference surface for height systems such as orthometric heights. It is a physical surface which represents the size and shape of the earth, by describing origin surfaces for point heights, determining mean earth ellipsoid, determining the horizontal and vertical datum of reference systems, examining changes in the earth and sea surfaces (Yilmaz and Karaali, 2010).

In this context, its physical realization is usually the mean sea surface as determined by ocean tide gauges (Hofmann and Moritz, 2006). In sum, the geoid surface is the closed surface going under the land which coincides with stable sea surface that is free of effects like temperature, pressure, density, salinity differences, currents and tides, and it is defined by its potential value (Yilmaz and Karaali, 2010). The geoids' height, N can therefore be defined as the separation of the ellipsoid surface with the geoid surface measured along the ellipsoidal normal as illustrated in Figure 1.

Geoid heights from the Global Geopotential Models (EGM96 and EGM08) are given as a set of spherical harmonic coefficients (Pavlis et al, 2008). Different datasets are often used to determine these coefficients ranging from satellite observations, which give the so-called satellite-only solutions, to data which incorporate satellite altimetry and surface gravity data (Rapp, 1996). For the African Geoid Project model (AGP03), geoid height is derived from the combination of Stokes's formula and the geopotential coefficients implied by EGM96 model (Merry, 2003).

2.2. Ellipsoid height

The ellipsoid is a geometric surface which approximates the geoid in a least squares sense. For geodetic purposes, the ellipsoid of revolution is produced when an ellipse is rotated about its semi minor axis, provides a well-defined mathematical surface whose shape and size are defined by two parameters viz., Size of a reference ellipsoid can be described by semi-minor axis, **b** or semi-major axis, **a;** Shape of a reference ellipsoid can be described by its flattening, **f** or its eccentricity, **e**.

The ellipsoid surface as a regular surface can be determined mathematically. It is for this reason that, as a reference surface, it is widely used for horizontal co-ordinate computations. Nevertheless, it is traditionally taken to be of limited use in heights as it ignores the flow of liquids (Hofmann & Moritz, 2006). The ellipsoid, h above the surface of ellipsoid is shown in Figure 1.

2.3. Orthometric height

Orthometric heights (H) are more desirable, because they better relate to mean sea level in the geophysical sense. Orthometric height refers to a vertical datum that is usually taken to be a best fit to mean sea level, either in a global sense or simply adopted from a local tide gage. Such a surface of equal potential of gravity (geopotential) best serves for describing height changes, because water will flow and self-level to the lowest geopotential surface (Roman et al., 2010). Theoretically, both ellipsoidal

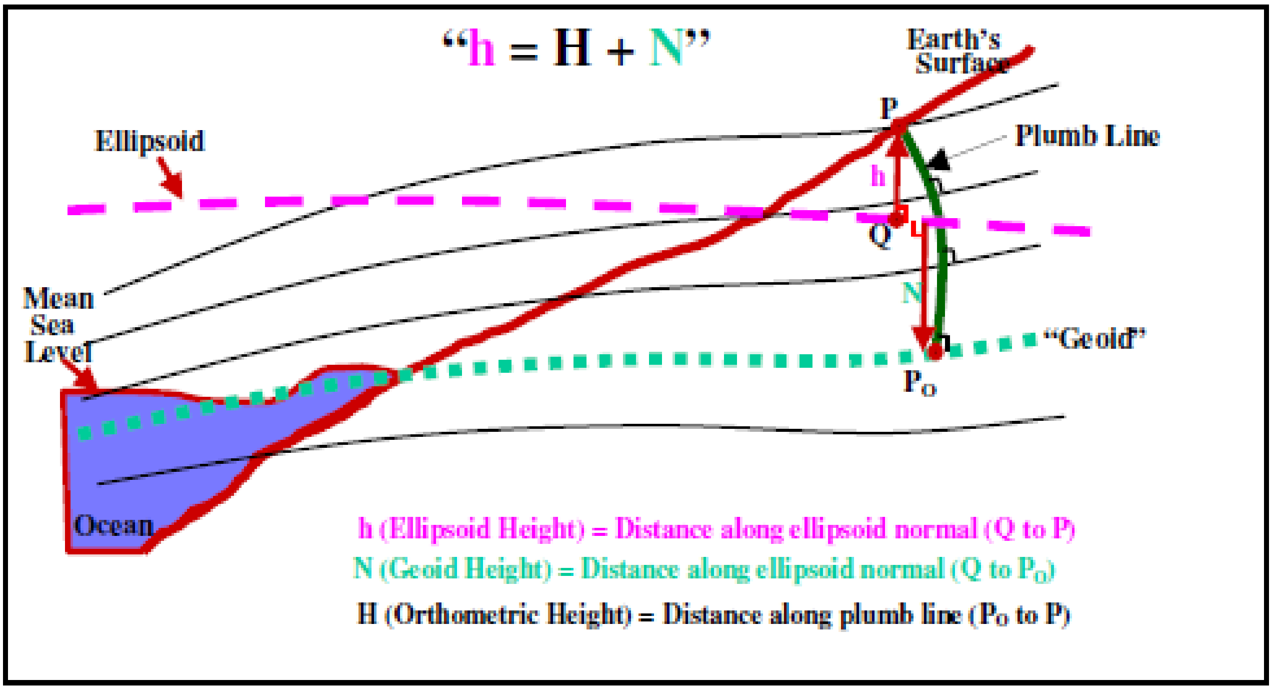

Figure 1: Relationship between geoid, ellipsoid and orthometric heights (Source: Roman et al., 2010)

height and orthometric heights are measured along the normal to the ellipsoid and along the direction of the plumb line (vertical), respectively. The actual gravity plumb line (over exaggerated in drawing) along which H is reckoned is a curved distance due to effects of direction of gravity, known as deflection of the vertical. For engineering purposes, the error produced by this approximation can normally be ignored. Figure 1 provides the relationship between geoid, ellipsoid and orthometric heights.

2.4 TPLN Orthometric heights

The Tanzania Primary Levelling Network (TPLN) was designed in the 1960 and implemented between 1961 and 1964 (Mayunga, 2016). It is comprised of 53 fundamental benchmarks (FBM) made up on loops based on local Mean Sea Level. The measurements were made on land in such a way that the misclosures between forward and back leveling between successive benchmarks is less than ± 3mm√k where 'k' is the leveled distance in kilometers. The distribution of the misclosures of the levelling data in the loops was done loop-wise after the completion of observations on each loop. At present the TPLN consist of eight (8) loops namely, loop A, B, C, D, E, F, G and H. The leveled orthometric heights in the TPLN are corrected for gravity effects on the basis of the normal gravity computed by means of the International Gravity Formula, 1930 (Deus, 2007). The establishment of the TPLN was referred to tide gauge measurements at the Tanga harbour whose mean sea level was used as a reference. The value for the mean sea level (MSL) at Tanga harbour was deduced from tide gauge readings taken during a 28 months' period from August 1962 to November 1964, both months inclusive. The MSL was used to determine the elevation of the Reference Fundamental Benchmark at Maweni. The other in land benchmarks were connected to the Maweni FBM through the observations of loop 'A' and the other loops of TPLN.

3. Materials and methods

The Weighted Mean Method was devoted to validate the positional accuracy of the model, the height accuracy or both positional and height accuracy of the models in absolute sense. In order to determine the Global Geoid Model that best fit with the GPS/Leveling in Tanzania, the geoid heights from EGM08, EGM1996, AGP03 and the TZG13 models were calculated. Generally, the EGM08 model incorporates satellite data (GRACE), terrestrial gravity data and altimetry data (Pavlis et al., 2008). The accuracy of the EGM08 equating to a degree and order 2160 model is claimed to be ± 15 centimeters worldwide. The EGM96 model that incorporates surface gravity data, altimeter-derived free air gravity anomalies from ERS-1 and from the GEOSAT Geodetic Mission Global Positioning System (GPS) data, NASA's Tracking and Data Relay Satellite System (TDRSS), the French DORIS system, and the US Navy TRANET Doppler tracking system as well as direct altimeter ranges from TOPEX/POSEIDON (T/P) and the accuracy for the EGM96 geoid was ± 50cm worldwide (Lemoine et al., 1998; Pavlis et al., 2008). The AGP03model incorporates terrestrial gravity data filled at the 5' grid terrestrial gravity data set using gravity anomalies implied by the EGM96 model, with same accuracy as of EGM96 (Merry, 2003) and the TZG13 that involves the use of spherical harmonic expansion of the Earth's geo-potential derived from the GRACE satellite mission (Ulotu, 2009). The TZG13 model has an accuracy of 29.7cm.

The EGM08 geoid heights used were obtained at 5' x 5' grid values for the area of interest and those computed directly on benchmarks from the geopotential coefficients using software supplied with the model. The EGM96 geoid height at each of the 13 benchmarks was obtained at 5' x 5' grid values by using NIMA EGM96 calculator program for Windows 95/NT downloaded from http://earthnfo.nima.mil/GandG/wgs84/gravitymod/egm96/egm96.html. The EGM96 geopotential coefficients using spherical harmonic representations by the following expansion that is complete to degree 360 was used to compute the geoid heights. The data files for the AGP03 geoid heights were downloaded at 5' x 5' gridded free air gravity anomalies.

3.1. Position of selected benchmarks on part of the TPLN

GPS observation was done at 13 benchmarks. The 13 points of GPS observation were made on TPLN as possible in accordance with latitude, longitude and ellipsoid heights as indicated in Figure 2. The geodetic coordinates (ϕ, λ, h) data in ITRF2005 at the 13 TPLN benchmarks from the processing of GPS data and the correction values (σ) were collected as illustrated in Table 1.

Table 1: ITRF2005 data based on curvilinear coordinates and their accuracies with published orthometric heights on 13 benchmarks from the TPLN

Benchmark Name	Geodetic coordinates	Precision (σ) in meters
FBM Dar-Aux	Latitude: -06 46 42.041181	0.0039
	Longitude: 39 17 07.006124	0.0052
	Ell. Height: -15.510m	0.0091
	TPLN Height: 11.012m	-
FBM Kwala	Latitude: -06 47 54.041181	0.0039
	Longitude: 38 34 42.510094	0.0052
	Ell. Height:54.951m	0.0091
	TPLN Height:79.983m	-
IBM3/54_kilosa-Aux	Latitude: -06 49 53.346951	0.0080
	Longitude: 36 59 09.370195	0.0055
	Ell. Height:469.00m	0.0102
	TPLN Height:489.145m	-
IBM5/47_Dodoma-Aux	Latitude: -06 11 00.250444	0.0034
	Longitude: 35 44 50.885173	0.0023
	Ell. Height:1112.557m	0.0047
	TPLN Height:1131.415m	-
FBM Kondoa	Latitude: -04 54 06.245608	0.0020
	Longitude: 35 48 31.521179	0.0035
	Ell. Height:1373.695m	0.0060
	TPLN Height:1391.300m	-
FBM Tabora	Latitude: -05 01 55.066958	0.0067
	Longitude: 32 49 12.393679	0.0043
	Ell. Height:1218.728m	0.0079
	TPLN Height:1235.726m	-
FBM Shinyanga	Latitude: -03 40 13.552474	0.0019
	Longitude: 33 26 02.314643	0.0042
	Ell. Height:1103.618m	0.0064
	TPLN Height:1122.014	-
FBM Mwanza-Aux	Latitude: -02 31 22.411311	0.0051
	Longitude: 32 53 51.559420	0.0033
	Ell. Height:1122.052m	0.0092
	TPLN Height:1138.444m	-
FBM Makuyuni	Latitude: -03 33 12.367130	0.0042
	Longitude: 36 05 49.065398	0.0061
	Ell. Height:1051.145m	0.0095
	TPLN Height:1069.00	-
IBM A 24/5_Moshi-Aux	Latitude: -03 22 45.000000	0.0033
	Longitude: 37 19 22.800000	0.0028
	Ell. Height:787.442m	0.0008
	TPLN Height:805.051m	-
IBM Korogwe-Aux	Latitude: -05 09 58.796850	0.0018
	Longitude: 38 27 38.421593	0.0030
	Ell. Height: 276.375m	0.0048
	TPLN Height: 298.854m	-
FBM Maweni	Latitude: -05 07 12.424007	0.0046
	Longitude: 39 00 49.658730	0.0058
	Ell. Height: 37.447m	0.0079
	TPLN Height:63.237m	-
IBM 15/31_ Wami-Aux	Latitude: -06 12 43.842551	0.0012
	Longitude: 38 42 44.531514	0.0015
	Ell. Height: -12.316m	0.0028
	TPLN Height: 13.020m	-

Table 2: GPS/levelling derived geoid heights (m) at the selected 13 TPLN benchmarks from the GPS measurements.

Benchmark Name	Ell. Height, h (m)	Levelled Height, H_{SMD} (m)	$N_{GPS} = h - H_{SMD}$ (m)
FBM Dar-Aux	-15.510	11.012	-26.522
FBM Kwala	54.951	79.983	-25.032
IBM3/54_kilosa-Aux	469.00	489.145	-20.145
IBM5/47_Dodoma-Aux	1112.557	1131.415	-18.858
FBM Kondoa	1373.695	1391.300	-17.605
FBM Tabora	1218.728	1235.726	-16.998
FBM Shinyanga	1103.618	1122.014	-18.396
FBM Mwanza-Aux	1122.052	1138.444	-16.392
FBM Makuyuni	1051.145	1069.00	-17.855
IBM A 24/51_Moshi-Aux	787.442	805.051	-17.609
IBM Korogwe- Aux	276.375	298.854	-22.479
FBM Maweni	37.447	63.237	-25.79
IBM 15/31_ Wami-Aux	-12.316	13.020	-25.336

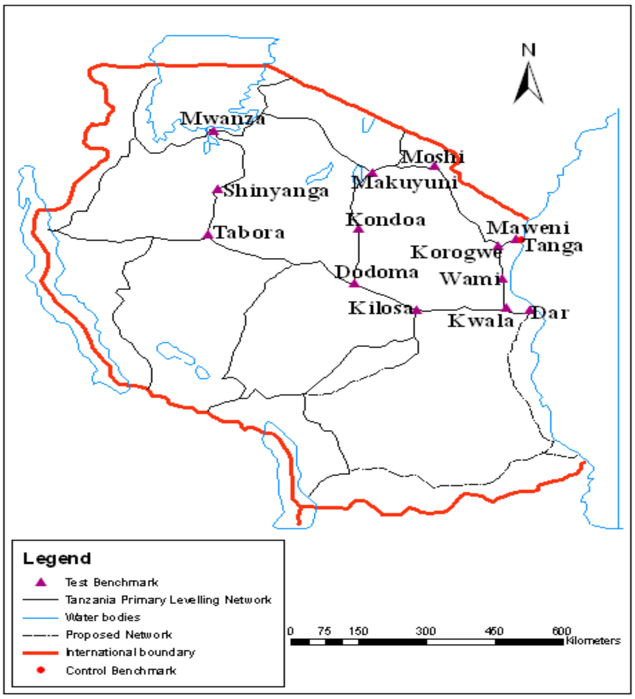

Figure 2: Selected 13 benchmarks for validation of Geoid Models from TPLN

4. Computations and discussion of results

4.1 GPS/Levelling-derived geoid heights (N_{GPS})

The geoid heights from GPS - derived ellipsoidal heights and the TPLN orthometric heights are referred to as GPS/levelling. The GPS/levelling geoid height results are shown in Table 2. The Pro-fix "Aux" after a benchmark name indicates that an auxiliary point had to be established to enable GPS observations.

4.2. Prediction of 5'x 5' geoid heights from geoid models

Predictions of geoid heights on the 13 TPLN benchmarks by using Weighted mean Method was done. The Weighted Mean approach is the method that makes the use of the weighted functions which reflect the fact that data points closer to the prediction points contribute to the accuracy of the value of the predicted geoid heights more than the distant ones. The method has proved to be economical and sufficiently accurate (John and Ulotu 2009, personal communication). The prediction is handled according to point-wise approach using the formula:

$$N_P = \frac{\left[\sum_{i=1}^{n} (N_i W_i)\right]}{\left[\sum_{i=1}^{n} W_i\right]} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}$$

The weight W_i is the reciprocal of the distance between point P and the corner points, N_i (i = 1, 2, 3, 4) such that:

$$\left.\begin{aligned} W_i &= L_i^{-1} \\ L_i &= \left(x_i^2 + y_i^2\right)^{1/2} \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(2)}$$

Where, x_i and y_i are the rectangular Cartesian coordinates of point i with P as the origin.

The predicted geoid heights (N_P) from the three geoid models denoted as $N_{EGM\,08-P}$, $N_{EGM\,96-P}$ and $N_{AGP\,03-P}$ were determined respectively. The given points were geoid heights at 5' x 5' grid coordinates around each benchmark.
The scheme for obtaining the geoid height of a benchmark P which lies within a 5' x 5' grid cell is depicted in Figure 3. The symbols N_1, N_2, N_3 and N_4 in Figure 3 denote geoid heights at the grid intersections (ϕ_i, λ_i) where $i = (1,2,3,4)$.

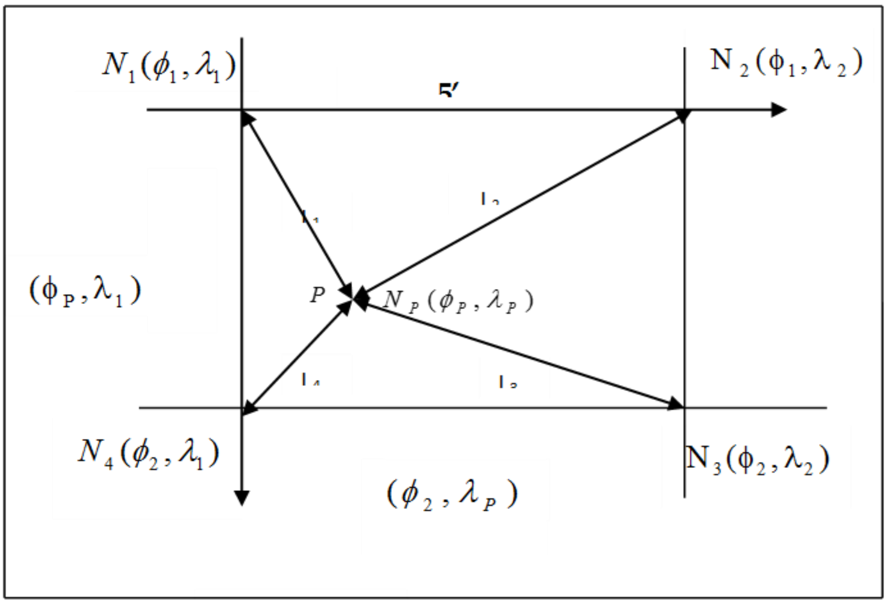

Figure 3: Prediction boundaries of 5' x 5' geoid height grids at all selected benchmarks using Weighted Mean Approach.

These predicted geoid heights were later denoted as N_{EGM08}, N_{EGM96}, N_{AGP03} and N_{TZG13}. For the purpose of geoid heights validation results, the predicted geoid heights from the EGM08, EGM96, AGP03 and the TZG13 geoid models respectively were computed and the results are shown in Table 3.

4.3. Computations and comparison of predicted geoid models against GPS/levelling geoid heights

Computations of geoid heights from Geoid Models were done. The differences of geoid heights from the GPS/levelling derived geoid heights and those predicted from the geoid models at co-located benchmarks provided discrete geometric control validation as illustrated in Table 4.

4.4. Summary of geoid heights differences

Table 5 shows the summary of the predicted geoid height differences results from the four geoid models (two being the global and others being local) at maximum, minimum, mean, root mean square as well as standard deviation respectively.

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Table 3: Predicted geoid heights results from EGM08, EGM96, AGP03 and the NTZG13 Geoid Models

Benchmark Name	**Predicted Geoid heights (Units in meters)**			
	$N_{EGM08-P}$	$N_{EGM96-P}$	$N_{AGP03-P}$	$N_{TZG13-P}$
FBM Dar-Aux	-27.755	-28.136	-27.364	-27.306
FBM Kwala	-26.163	-26.649	-25.272	-25.708
IBM 3/54_Kilosa – Aux	-21.255	-21.486	-19.924	-19.091
IBM 5/47_Dodoma-Aux	-20.182	-20.293	-19.126	-19.517
FBM Kondoa	-18.968	-19.157	-18.082	-18.192
FBM Tabora	-17.997	-19.035	-17.948	-18.078
FBM Shinyanga	-19.575	-20.095	-19.1998	-19.113
FBM Mwanza-Aux	-17.610	-17.792	-16.874	-16.914
FBM Makuyuni	-19.018	-19.218	-18.352	-18.549
IBM A 24/51_Moshi-Aux	-18.641	-17.098	-18.165	-18.136
IBM 27/55 Korogwe-Aux	-23.518	-23.921	-22.622	-23.031
FBM Maweni	-26.944	-27.589	-26.296	-26.305
IBM Wami	-26.728	-26.918	-25.506	-26.337

Table 4: GPS/levelling-derived geoid heights against the predicted geoid heights from the four Geoid Models

Benchmark Name	Geoid Heights from GPS/Levelling	Geoid Heights from Models				Differences in Geoid Heights			
	N_{GPS}	N_{EGM08}	N_{EGM96}	N_{AGP03}	N_{TZG13}	N_{GPS}-N_{EGM08}	N_{GPS}-N_{EGM96}	N_{GPS}-N_{AGP03}	N_{GPS}-N_{TZG13}
FBM Dar	-26.522	-27.755	-28.136	-27.364	-27.306	1.233	1.614	0.842	0.784
FBM Kwala	-25.032	-26.163	-26.649	-25.272	-25.708	1.131	1.617	0.240	0.676
IBM 3/54_Kilosa –Aux	-20.145	-21.255	-21.486	-19.924	-19.091	1.110	1.341	-0.221	-1.054
IBM 5/47_Dodoma-Aux	-18.858	-20.182	-20.293	-19.126	-19.517	1.324	1.435	0.268	0.659
FBM Kondoa	-17.605	-18.968	-19.157	-18.082	-18.192	1.363	1.552	0.477	0.587
FBM Tabora	-16.998	-17.997	-19.035	-17.948	-18.078	0.999	2.037	0.950	1.080
FBM Shinyanga	-18.396	-19.575	-20.095	-19.1998	-19.113	1.179	1.699	0.804	0.717
FBM Mwanza-Aux	-16.392	-17.610	-17.792	-16.874	-16.914	1.218	1.400	0.482	0.522
FBM Makuyuni	-17.855	-19.018	-19.218	-18.352	-18.549	1.163	1.363	0.497	0.694
IBM A 24/51_Moshi-Aux	-17.609	-18.641	-17.098	-18.165	-18.136	1.032	-0.511	0.556	0.527
IBM Korogwe-Aux	-22.479	-23.518	-23.921	-22.622	-23.031	1.039	1.442	0.143	0.552
FBM Maweni	-25.790	-26.944	-27.589	-26.296	-26.305	1.154	1.799	0.506	0.515
IBM Wami	-25.336	-26.728	-26.918	-25.506	-26.337	1.392	1.582	0.170	1.001

5. Discussion

The results obtained in this research shows that there are differences between GPS/levelling derived geoid heights and those obtained from the four geoid models. The geoid height differences from the GPS/levelling geoid heights and those from EGM08 model range from 0.999 m to 1.392 m. The geoid height differences from EGM96 model and GPS/levelling geoid heights ranges from -0.511 m to 2.037 m while the geoid height differences from AGP03 model against GPS/levelling geoid heights range from -0.221 m to 0.950 m. The differences from TZG13 model versus GPS/levelling geoid heights range from -1.054 to 1.080 m. Figure 4 depicts graphically the geoid height differences from the geoid models based on the standard GPS/levelling derived geoid heights in Tanzania. Table 5 provides summary of predicted geoid height differences.

Table 5: Summary of the predicted geoid height differences results

Geoid Models	Min	Max	Mean	RMS	SD= s
EGM08(Nmax = 2190)	0.999	1.392	1.192	± 1.186	± 0.120
EGM96(Nmax = 360)	-0.511	2.037	1.402	± 1.530	± 0.586
AGP03	-0.221	0.950	0.440	± 0.538	± 0.310
TZG13	-1.054	1.080	0.558	± 0.747	± 0.717

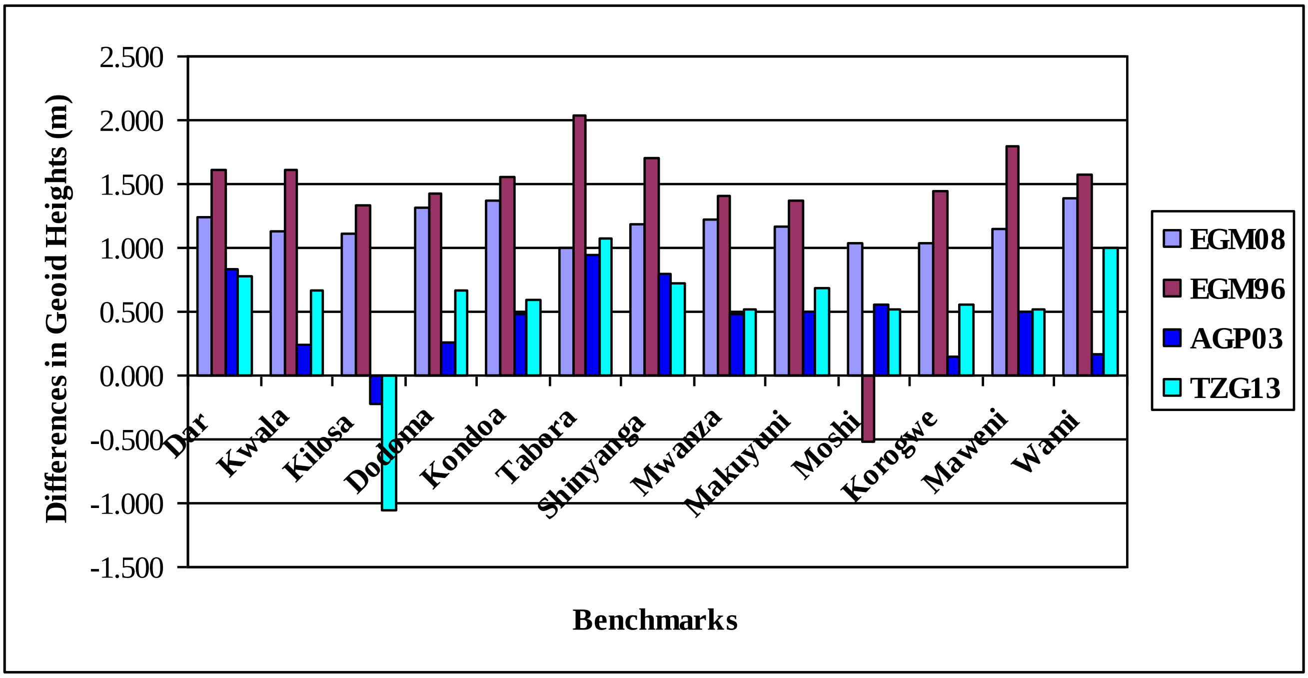


Figure 4: Comparisons of absolute differences of the predicted geoid heights from each model based on GPS/levelling derived geoid heights

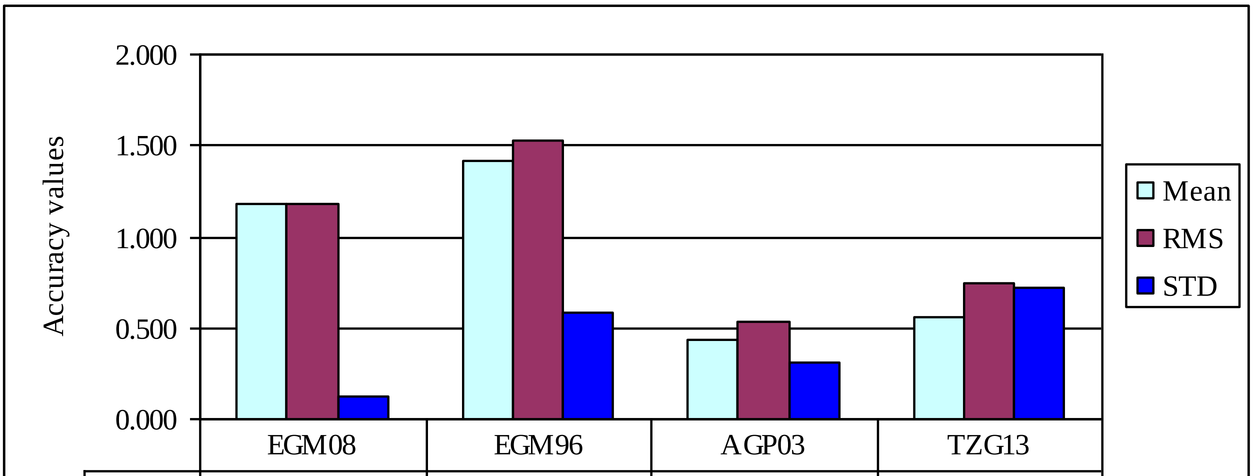
The mean differences are 1.180 m, 1.413 m, 0.440 m and 0.558 m; the RMS differences are ±1.186 m, ±1.530 m, ± 0.540 m and 0.747 m; the sample standard deviations are ± 0.120 m, ± 0.586 m, ±0.310 m and 0.717 from EGM08, EGM96, AGP03 and TZG13 respectively as represented graphically in Figure 5.

6. Conclusions

The results of these comparisons of the Geoid models against the GPS-Levelling derived geoid heights over 13 TPLN benchmark shows that among the developed geoid models EGM08, EGM96, AGP03 and the TZG13, the AGP03 model is the best geoid model that fits the GPS and levelling data in Tanzania at present.

The mean differences of 0.440 m and the RMS of ± 0.538 m lead to the conclusion that the AGP03 geoid model is a better model for GPS/Levelling in Tanzania than other three geoid models.

Thus the EGM08 geoid model does not produce geoid heights that are closer to GPS/levelling geoid heights; rather the AGP03 geoid model does so. The major contribution may however come from the higher wavelength of 5’ which improves the consistence



	EGM08	EGM96	AGP03	TZG13
Mean	1.180	1.413	0.440	0.558
RMS	1.186	1.530	0.540	0.747
STD	0.120	0.586	0.310	0.717

Figure 5: Accuracy evaluation of geoid models due to comparisons at independent test benchmarks in the area of interest

between the AGP03 and the GPS/levelling geoid heights such that its shorter wavelength of 5' during its computations and the fact that Stoke's formula is more sensitive to short wavelength components of geoid heights than the higher degree (> 360) spherical harmonics in the EGM08. Thus, the results obtained from point-wise validation have revealed that the AGP03 geoid model performs exceedingly better than other models over the area of interest. The TZG13 which was expected to be the best model as tested in the 13 TPLN benchmarks, does not provide the best accuracy as compared to AGP03.

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