

## Modelling uncertainties in differential global positioning system dataset

I. Yakubu<sup>1\*</sup> and I. Dadzie<sup>2</sup>

<sup>1</sup>Department of Geomatic Engineering, University of Mines and Technology, Tarkwa, Ghana

<sup>2</sup>Department of Geomatic Engineering, Kwame Nkrumah University of Science and Technology, Kumasi, Ghana

\*Email: [yissaka@umat.edu.gh](mailto:yissaka@umat.edu.gh)

(Received: Jan 18, 2019; in final form: May 07, 2019)

**Abstract:** The quest for precision and accuracy in Differential Global Positioning System (DGPS) data requires modelling uncertainties in the data collected and eliminating outliers which hinder the precision and accuracy. In this paper, dual frequency DGPS receivers' dataset collected repeatedly over a control station at thirty (30) minutes interval were mathematically modelled for uncertainties using five soft computing and classical methods namely: Backpropagation Artificial Neural Networks (BPANN), Generalised Regression Neural Networks (GRNN), Multivariate Adaptive Regression Splines (MARS), Radial Basis Functions Neural Network (RBFNN), and Total Least Square. The results revealed that all the models produced were satisfactory. The Mean Horizontal Error (MHE), Root Mean Square Error (RMSE), and Standard Deviation (SD) performance criteria indices were applied. GRNN outperformed BPANN, MARS, RBFNN, and TLS in modelling DGPS data uncertainties. In terms of their mean horizontal displacement and standard deviation, GRNN achieved 4.5314E-11 m and 1.3200E-13 m compared to TLS, BPANN, MARS and RBFNN which achieved: 7.3901E-06 and 8.7500E-14; 2.8311E-06 and 2.2300E-08; 0.0088 m and 3.3158E-05 m; and 1.2016E-04 m and 1.2195E-06 m respectively. It can be concluded that all the models used can be applied in detecting and eliminating uncertainties in DGPS data. There is therefore, the need to apply these methods in modelling uncertainties in DGPS applications in sensitive areas such as deformation monitoring of high rise buildings, bridges and dam embarkment.

**Keywords:** Data Uncertainties, Soft Computing, Regression Techniques, DGPS

### 1. Introduction

In recent years, the use of Global Positioning System (GPS) for positioning has gained more grounds than the traditional surveying techniques which required the use of a theodolite or total station. Schuessler and Axhausen (2009), conducted a research and concluded that Differential Global Positioning Systems (DGPS) has provided surveyors with more accurate results than the traditional techniques. The use of the DGPS help pinpoints satellite locations of interest. Most of the GPS data has to be processed before becoming meaningful to the user. Hence, scientists and engineers find the term post-processed GPS data appropriate to describe GPS data which is not processed directly on the field, rather obtained after some processing in the office.

Nonetheless, the use of the GPS receivers come with some restrictive conditions including hazy weather conditions, shade, an insufficient number of required satellites for positioning. These conditions cause a delay in obtaining position information of the observation point thereby causing long observational periods and introduce errors into the observed data.

These errors, classified as either parametric or non-parametric, need to be removed or minimized with the use of statistical tools such as the box and whisker plot (box plot) and the modified z-score which could be used to identify outliers in a given dataset (Ben-Gal, 2005). Outliers are blunders which are committed on the field and are non-adjustable. They have to be eliminated from datasets since they are either the very small or big values identified during data analysis (Carlson and Goodman, 2014). On the other hand, systematic errors which are usually fixed and have a mean of zero (0) follow a pattern, and hence are predictable. Random errors are considered

the remaining errors after all other errors have been removed (Filzmoser, 2004).

Least square regression models invented by Gauss (1823) have been applied for solving majority of problems in geoscientific field, notably adjustment of DGPS survey networks (Yakubu *et al.*, 2018; Peprah and Mensah, 2017; Ansah, 2016; Annan *et al.*, 2016; Okwuashi and Eyoh, 2012), determination of GPS coordinate transformation parameters (Ziggah *et al.*, 2013), and datum transformation parameters (Ziggah *et al.*, 2016) with the purpose of identifying and eliminating the uncertainties in the datasets. However, (Acar *et al.*, 2006) noted that some outliers in some datasets still remain after adjustments mainly due to inefficiencies of the mathematical models to eliminate the outliers. This study, therefore, seeks to investigate more sophisticated and advanced models for data pruning, denoising and eliminating outliers or uncertainties in DGPS datasets.

The invention of soft computing techniques has revolutionized data pruning and adjustment mainly because they have the capabilities to denoise datasets (Kutoglu, 2006) and give better accuracy in points estimation (Akyilmaz *et al.*, 2009) and have been applied to solve numerous scientific problems including improving classification (Sharma *et al.*, 2015; Kumar *et al.*, 2014), adjusting DGPS networks (Yakubu *et al.*, 2018), modeling stream networks (Achour *et al.*, 2012), earthquake modelling (Pinho *et al.*, 2008), landslide modelling (Zahra, 2010), coordinate transformation (Ziggah *et al.*, 2016), GPS height conversion (Fu and Liu, 2014; Liu *et al.*, 2011), geodetic deformation modelling (Bao *et al.*, 2011; Du *et al.*, 2014; Maxime *et al.*, 2005), earth orientation parameter determination (Liao *et al.*, 2012), estimating energy demand (Alreja, *et al.*, 2015), slope stability analysis (Samui, 2013; Lall *et al.*, 1996).

In this study, soft computing techniques such as Backpropagation Artificial Neural Networks (BPANN), Generalised Regression Neural Networks (GRNN), Radial Basis Functions Artificial Neural Networks (RBFNN), and Multivariate Adaptive Regression Splines (MARS) were used in modelling the uncertainties in DGPS datasets. The performances of these soft computing techniques were compared with classical techniques such as Total Least Squares (TLS) which has the capability to adjust errors in both the observation and design matrices (Acar *et al.*, 2006), and recommendations made.

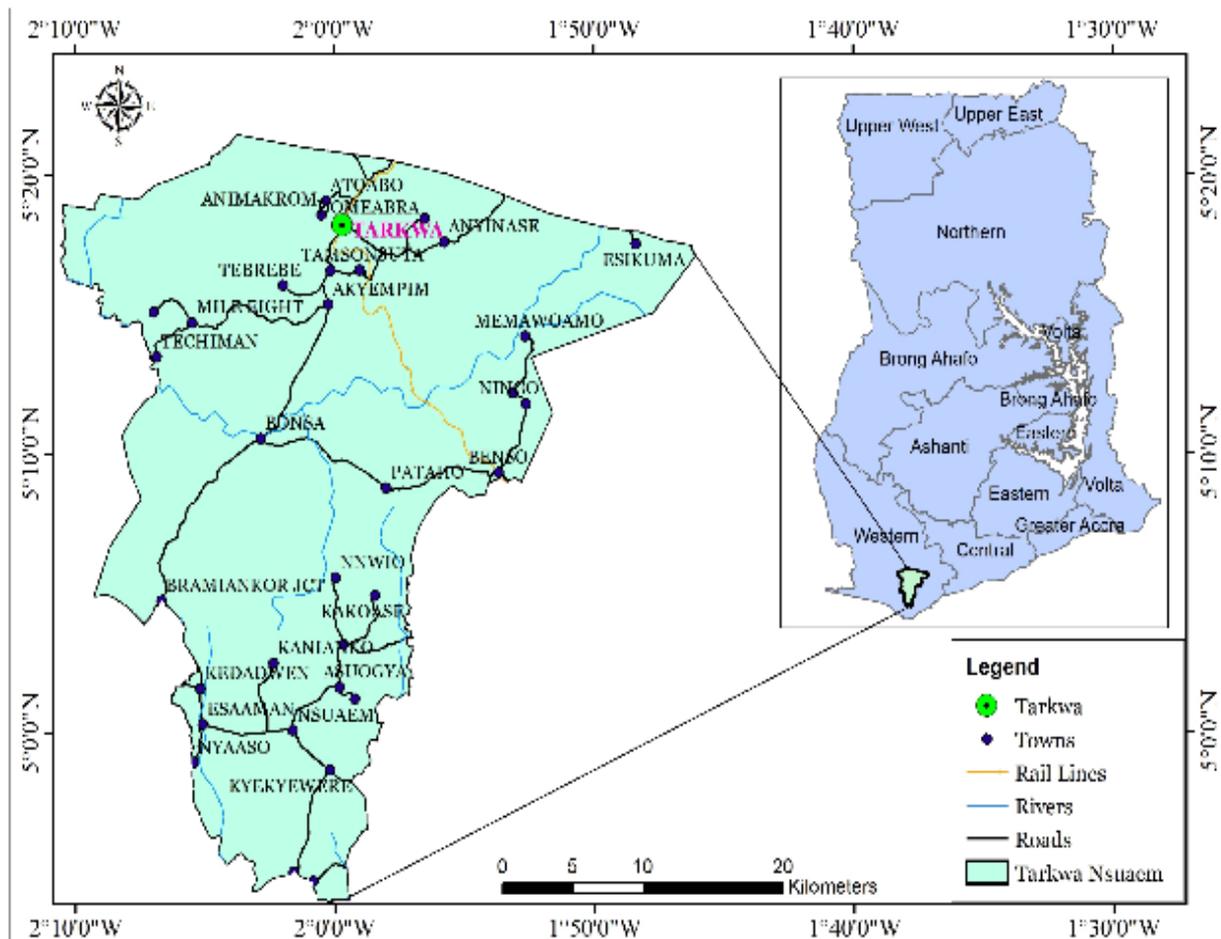
**2. Resources and methods used**

Tarkwa is the study area (Figure 1) which located in the Western Region of the Republic of Ghana (Asklunel and Eldvall, 2005) with geographic coordinates between longitude 2° 10' 00" W - 1° 45' 00" W and latitude 4° 30' 00" N - 5° 25' 00" N with an average topographic height of about 78 m above Mean Sea Level (MSL). Geographically, the topography is generally undulating with steep slopes parallel to each other and to the strike of the rocks in the north-south direction (Kortatsi, 2004). The type of coordinate system used in the study area is Ghana projected grid derived from the Transverse Mercator with 1° W Central Meridian and the WGS84 (UTM Zone 30N)

(Yakubu *et al.*, 2018; Peprah *et al.*, 2017). Tarkwa is a mining town, with three major mining companies, namely: Goldfields Ghana Limited, Tarkwa Mine; AngloGold Ashanti, Iduaprim Mine and Ghana Manganese Company Limited, Nsuta.

DGPS survey was carried out on a known control in Tarkwa, the study area. The DGPS field observations were conducted at 30-minute intervals over the same control to obtain one hundred and four (104) redundant coordinates data for the control. These observations were made to check the consistency of the output data for the same position. The base station was a Continuous Operating Reference Station (CORS) located at the University of Mines and Technology, Tarkwa.

One of the contributing factors affecting the estimation of accuracy is related to the quality of datasets used (Devi and Karthikeyan, 2015; Dreiseit and Ohno-Machado, 2002; Ismail *et al.*, 2012), several precautions including time of observation, avoiding overhead cables, multipath errors (Yakubu and Kumi-Boateng, 2011) were considered to ensure the reliability of the observed datasets. Table 1 shows sample DGPS dataset used in modelling the uncertainties



**Figure 1: Study area**

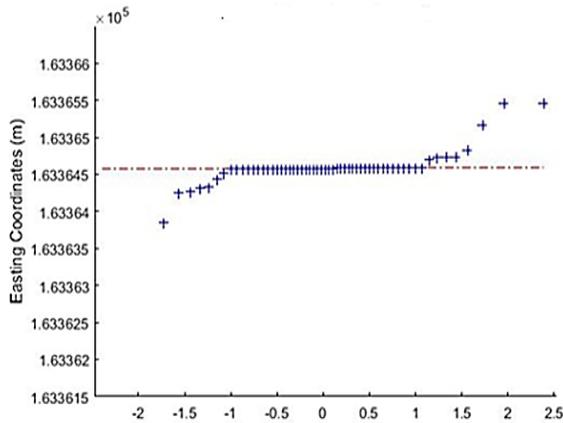
**Table 1: Sample of data used for the study (Units in meters)**

SN	Easting (E)	Northing (N)	Elevation (Z)	SN	Easting (E)	Northings (N)	Elevation (Z)
1	163364.5833	69577.1700	76.1957	11	163364.7337	69577.1290	76.3378
2	163364.5338	69577.1707	76.2743	12	163364.5816	69577.1695	76.1958
3	163364.2055	69577.2139	76.2182	13	163364.5829	69577.1701	76.1997
4	163364.3048	69577.3004	75.6254	14	163364.5169	69577.1430	76.1977
5	163364.4388	69577.2636	75.1524	15	163364.5776	69577.2190	75.7588
6	163364.5803	69577.1708	76.2003	16	163364.5784	69577.1706	76.1886
7	163365.1671	69576.8470	76.4418	17	163364.5882	69577.2844	76.6442
8	163364.3452	69576.9806	76.1168	18	163364.5821	69577.1674	76.1937
9	163364.5871	69577.1773	76.2827	19	163364.5820	69577.1680	76.1889
10	163364.5821	69577.1703	76.2027	20	163364.7380	69577.1282	75.8962

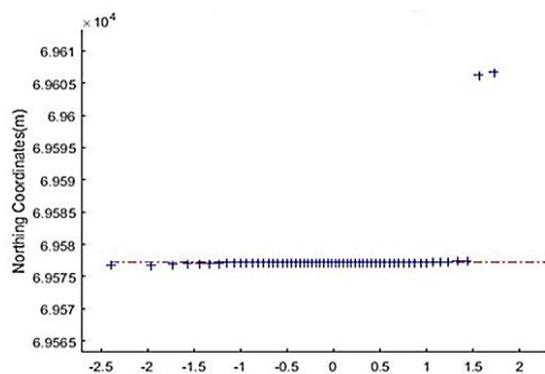
**2.1 Methods**

**2.1.1 Outlier Detection**

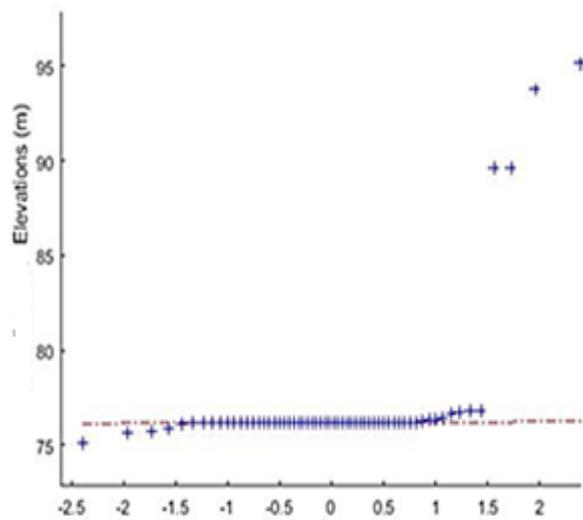
All DGPS data were filtered and cleaned for errors arising from dilution of precision, inadequate number of satellites during observations *etc.*, as advised by Schuessler and Axhausen (2009). Techniques like the Grubb’s Test, Tietjen-Moore Test, Generalised Extreme Studentized Deviate (ESD) Test (Anon., 2013) were applied to remove outliers. Probability plots of the eastings, northings and elevations were conducted to show their central tendencies as shown in figures 2, 3 and 4.



**Figure 2: Probability plot of easting coordinates**



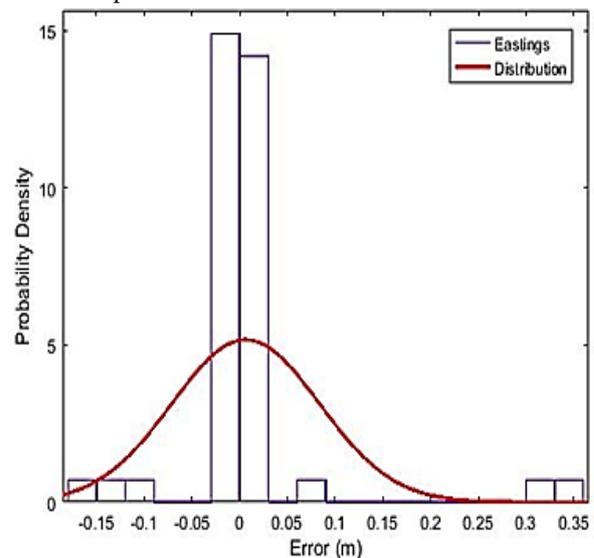
**Figure 3: Probability plot of northings coordinates**



**Figure 4: Probability plot of elevations**

*Error Fitting*

A Gaussian distribution fit was applied to the dataset to monitor the distribution of the errors. Figures 5, 6 and 7 show the plots of the error distribution.



**Figure 5: Error fitting for eastings**

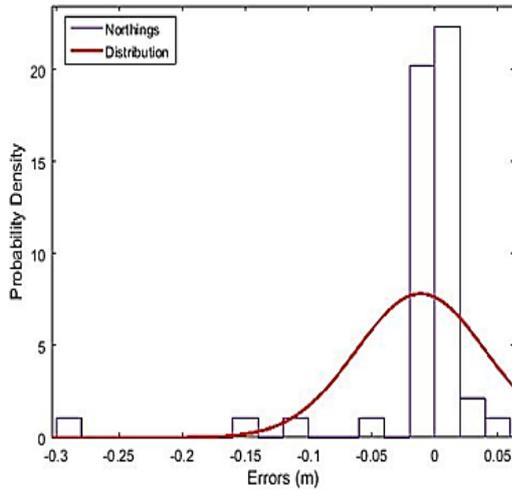


Figure 6: Error fitting for northings

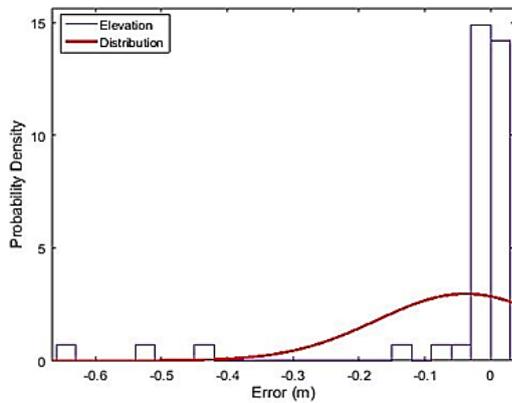


Figure 7: Error fitting for elevations

The fits were realized to be skewed with the summary statistics shown in table 2.

**Table 2: Summary statistics for error distribution curve**

Coordinate	Parameter		
	Skewness		Standard Deviation
X	-2.7795	2.7459	0.0764
Y	4.2465	-4.1948	0.0505
Z	3.5389	-3.4934	0.1330

### 2.1.2 Backpropagation Artificial Neural Network (BPANN)

The BPANN training algorithm involves three stages: the input layer, hidden layer and output layer. In this study, the input variables were the Eastings, Northings, and Elevations denoted as  $(E_{i,j}, N_{i,j}, Z_{i,j})$  and the output variables were the Eastings and Northings denoted as  $(E_{\text{output}}, N_{\text{output}})$ . In the BPANN model formulation, the dataset is normalized to ensure constant variation in the model. The selected input and output variables were normalized between the intervals  $[-1, 1]$  as presented by Equation (1) (Mueller and Hemond, 2013).

$$y_i = y_{\min} + \frac{(y_{\max} - y_{\min}) \times (x_i - x_{\min})}{(x_{\max} - x_{\min})} \quad (1)$$

where  $y_i$  denotes the normalized data;  $x_i$  is the measured GPS dataset values;  $x_{\min}$  and  $x_{\max}$  are the minimum and maximum values of the measured GPS dataset with  $y_{\max}$  and  $y_{\min}$  values set at 1 and -1, respectively.

Bayesian Regularization learning algorithm was used for the training to find the optimum weight combination. The datasets were divided into training (70 %) and testing (29 %). At the point where the error starts to increasing the training is stopped (Chakraborty and Goswami, 2017). The tansig and purelin activation functions were used for the hidden and output layer respectively in the network training. BPANN is an iterative training procedure, therefore the network was trained varying the number of hidden neurons ranging from 1 to 30 until the optimal model was achieved.

### 2.1.3 Radial Basis Function Neural Networks (RBFNN)

RBFNN model is an unsupervised learning algorithm and consist of three layers namely; an input layer, a hidden layer and an output layer (Ziggah *et al.*, 2016). The input layer is made up of sensory units that connect the network to its environment. In the second layer, the only hidden layer in the network applied a nonlinear transformation from the input space to the hidden space. The output layer is linear, supplying the response of the network to the activation pattern applied to the output layer. In this study, the input variables were the Eastings, Northings, and Elevations denoted as  $(E_{i,j}, N_{i,j}, Z_{i,j})$  and the output variables were the Eastings, Northings and Elevations denoted as  $(E_{\text{output}}, N_{\text{output}}, Z_{\text{output}})$ . The dataset used for the formulation of the model were divided as training data which consist of 60 % of the total dataset and testing data which consists of 29 %. RBFNN is an exact interpolator (Erdogan, 2009), hence a linear function is used in the input neurons and the connection between the input and hidden layers are not weighted (Kaloop *et al.*, 2017). In this presented study, the Gaussian function is applied, and the output neuron is a summation of the weighted hidden output layer given by Equation (2) (Erdogan, 2009) as

$$y(x) = \sum_{j=1}^n \kappa_j \chi_j(x) \quad (2)$$

where  $n$  is the number of hidden neurons,  $x \in R^M$  is the input,  $\kappa_j$  are the output layer weights of the radial basis function network,  $\chi_j(x)$  is Gaussian radial basis function given by Equation (3) as (Srichandan, 2012; Idri *et al.*, 2010):

$$\chi_j(x) = e^{\left( \frac{-\|x_i - c_j\|^2}{\sigma_j^2} \right)} \quad (3)$$

where  $c_j \in R^M$  and  $\sigma$  are the centre and width of  $j$ th hidden neurons respectively,  $\| \cdot \|$  denotes the Euclidean distance.

### 2.1.4 Generalized Regression Neural Networks (GRNN)

GRNN is a different kind of Radial Basis Function Neural Network (RBFNN) which is based on Kernel regression networks (Hannan *et al.*, 2010) with one pass learning algorithm and highly parallel structure (Dudek, 2011). It comprises of four layers namely; input layer, pattern layer (radial basis layer), summation layer, and an output layer. It was developed by Specht (1991). In this study, GRNN is being adopted and applied on DGPS dataset to model uncertainties. The input variables were the Eastings, Northings, and Elevations denoted as  $(E_{ij}, N_{ij}, Z_{ij})$  and the output variables were the Eastings and Northings denoted as  $(E_{output}, N_{output})$ . The number of input units in the first layer depends on the total number of the observational parameters. The first layer is linked to the pattern layer and in this layer, each neuron is being presented by a training pattern and its output. The pattern layer is connected to the summation layer. The summation layer consists of two different types of summation namely, single division unit and summation unit (Hannan *et al.*, 2010). The summation with output layer combined perform a normalization of output datasets. In training of the network, radial basis and linear activation functions are used in hidden and output layers. Each pattern layer unit is connected to two neurons in the summation layer. One neuron unit computes the sum of the weighted response of the pattern, and the other neuron unit computes unweighted outputs of pattern neurons. The output layer divides the output of each neuron unit by each other yielding the predicted output variables (Equation (4)):

$$y_i = \frac{\sum_{i=1}^n y_i \cdot \exp - G(x, x_1)}{\sum_{i=1}^n \exp - G(x, x_1)} \quad (4)$$

where  $y_i$  is the weighted connection between the  $i$ th neuron in the pattern layer and the summation neuron,  $n$  is the number of training patterns,  $G$  is the Gaussian function given by Equation (5) as

$$G(x, x_1) = \sum_{(k=1)}^m \frac{(x_1 - x_{1k})^2}{\sigma} \quad (5)$$

where  $m$  is the number of elements of an input vector,  $x_l$  and  $x_{lk}$  are the  $j$ th elements of  $x$  and  $x_i$  respectively. During the network training, the spread parameter was varied between 0 and 1 until the output with minimal residuals in terms of statistical analysis was achieved. This same procedure was also done when training the RBFNN.

### 2.1.5 Multivariate Adaptive Regression Splines (MARS)

The MARS is nonparametric regression technique which works by dividing the variables into regions, producing each region as a least squares equation (Friedman, 1991;

Leathwick *et al.*, 2005). Unlike Ordinary Least Squares, MARS assumes no functional relationship between the target and the predictor variables. The MARS model employed in this study is adopted from the work of Samui and Kurup (2012). The estimation of MARS model is developed in two steps. In the first step (the forward algorithm), MARS is estimated with an excessive number of knots in order to get a better estimate of the predictor variable (Samui and Kim, 2012). In the second step, the knots that contribute significantly to the overall estimation are retained whiles eliminating the less significant once. To ensure the goodness of fit, the Generalized Cross-Validation (GCV) is use to remove the redundant basis functions (Samui and Kothari, 2012; Craven and Wahba, 1979). In this present study, the Salford Predictive Model (SPM) software was adopted to train the MARS model (Yakubu *et al.*, 2018).

### 2.1.6 Total Least Square (TLS)

TLS is an iterative least squares estimation technique of determining the structure and estimating unknown parameters of a given model (Golub and Van Loan, 1980). In TLS, the orthogonal function estimates the model's parameters one at a time. Also, the percentage reduction with respect to the average of the squared residuals as well as the relative contribution of each term is estimated. Therefore, TLS takes into account the observational errors on both target and the predictor variables which in literature, gives more accurate results (Golub and Van Loan; 1980; Yanmin *et al.*, 2011). The implementation of TLS technique in this study is adopted from Yakubu *et al.* (2018). The adequacy of the estimated model will be tested to ascertain its overall fitness.

### 2.1.7 Models performance assessment

It is very essential to assess and validate the performance of each estimated model so as to establish whether the model is optimal. This study employs three of the frequently used Performance Indicators (PIs) such as the Root Mean Square Error (RMSE), Mean Horizontal Error (MHE), and Standard Deviation (SD) (Yakubu *et al.*, 2018). The selected PIs will be used in selecting the optimal model to represent each technique considered in this study. After the selection of the optimal models, the best of the techniques for modelling the uncertainties will be selected based on the technique (s) accounting for the least errors.

## 3. Results and discussion

Statistical methods and analysis were applied to prune the dataset in order to detect and eliminate outliers. The statistical analysis of the two-dimensional shift of the observed data are tabulated in table 3 and table 5. From the tables, it can be observed that the points gradually deviate from their true position. This can be due to the shift in the earth crust, delayed in the transmission of the propagated signal, or the type of the instrument used. This can cause serious problems in higher engineering works which requires a high degree of precision and accuracy.

**Table 3: Model results for soft computing techniques (units in meters)**

<b>BPANN (Eastings)</b>			
<b>PCI</b>	<b>ME</b>	<b>RMSE</b>	<b>SD</b>
Training	-8.8485E-08	3.0680E-06	8.0400E-09
Testing	-3.6570E-06	1.7798E-05	9.5000E-08
<b>BPANN (Northings)</b>			
<b>PCI</b>	<b>ME</b>	<b>RMSE</b>	<b>SD</b>
Training	3.2645E-07	2.5749E-06	5.1000E-09
Testing	5.2468E-08	8.5937E-07	9.1200E-09
<b>RBFNN (Eastings)</b>			
<b>PCI</b>	<b>ME</b>	<b>RMSE</b>	<b>SD</b>
Training	1.1974E-10	3.0754E-06	9.3500E-09
Testing	-3.9056E-04	1.9850E-03	1.3456E-05
<b>RBFNN (Northings)</b>			
<b>PCI</b>	<b>ME</b>	<b>RMSE</b>	<b>SD</b>
Training	1.0394E-12	3.0449E-07	3.2200E-10
Testing	1.4257E-05	5.4049E-05	3.6000E-07
<b>GRNN (Eastings)</b>			
<b>PCI</b>	<b>ME</b>	<b>RMSE</b>	<b>SD</b>
Training	2.0789E-12	4.9562E-11	8.1300E-13
Testing	8.0286E-12	4.5858E-11	3.4100E-12
<b>GRNN (Northings)</b>			
<b>PCI</b>	<b>ME</b>	<b>RMSE</b>	<b>SD</b>
Training	6.2365E-13	1.9755E-11	9.0400E-15
Testing	-5.0179E-12	1.9486E-11	1.2200E-12
<b>MARS (Eastings)</b>			
<b>PCI</b>	<b>ME</b>	<b>RMSE</b>	<b>SD</b>
Training	0.0012	0.0047	5.7164E-05
Testing	-0.0132	0.0742	0.0003
<b>MARS (Northings)</b>			
<b>PCI</b>	<b>ME</b>	<b>RMSE</b>	<b>SD</b>
Training	-0.0005	0.0024	1.9578E-05
Testing	0.0009	0.0023	7.4954E-05

The optimal equation for modelling the uncertainties in both the Eastings and Northings is given by Equation 6 and Equation 7 respectively. The basis functions used is tabulated in table 4.

$$E_i = 163364 + 1 \times BF1 + 1.98271e^{(-005)} \times BF2 + 2.0567e^{(-005)} \times BF6 \quad (6)$$

$$N_i = 69576.8 + 1 \times BF1 + 2.92989e^{(-005)} \times BF2 - 3.6429e^{(-005)} \times BF4 + 7.69176e^{(-006)} \times BF12 \quad (7)$$

**Table 4: Basis functions used by the MARS model**

<b>Eastings</b>	<b>Northings</b>
BF1 = max (0, E-163364);	BF1 = max (0, N-69576.8);
BF2 = max (0, E -163365);	BF2 = max (0, N-69577.2);
BF6 = max (0, E-163365);	BF4 = max (0, N-69577.2);
	BF12 = max (0, N-69577.2);

**Table 5: Results for all models (units in meters)**

<b>Model</b>	<b>MHE</b>	<b>RMSE</b>	<b>SD</b>
TLS	7.3901E-06	7.3901E-06	8.7500E-14
GRNN	4.5314E-11	5.2345E-11	1.3200E-13
RBFNN	1.2016E-04	1.0750E-03	1.2195E-06
BPANN	2.8311E-06	1.0215E-05	2.2300E-08
MARS	0.0088	0.0404	3.3158E-05

#### 4. Conclusions and recommendations

Outliers that remain uncertainties in datasets can affect the accuracy of evaluation procedures and estimated parameters if not eliminated. Several researchers have come up with many mathematical models such as classical least squares which have been used for decades. In recent times, with the advancement of science and technology, soft computing techniques have revolutionized the difficulties and deficiencies with the use of classical methods due to its capabilities in denoising datasets to yield a better estimate than the classical methods. This present study assessed the performance of soft computing techniques in modelling the uncertainties of DGPS dataset of two control stations whose coordinates are known to a certain degree of accuracy. The soft computing methods adopted were the BPANN, GRNN, MARS, and RBFNN. The study also compares the performance of these soft computing methods to classical methods such as the TLS due to the wide recommendation by researchers about its efficiency in modelling dataset to give a better estimate. The statistical analysis of the study reveals that all models gave satisfactory result in modelling the DGPS dataset. GRNN outperformed BPANN, RBFNN, MARS, and TLS in modelling the uncertainties in the dataset to give a better estimate. It can, therefore, be stated that all the methods can be used in modelling uncertainties in DGPS datasets.

But with respect to this research the GRNN model has demonstrated superiority over the other models. Hence, natural and engineered structures being monitored for deformation should take into account these methods for modelling the uncertainties in the dataset collected. This can also be automated and integrated in any deformation monitoring process.

This study does not only have a localised significance but will also open more scientific discourse into the applications of soft computing techniques in solving some of the problems in surveying and related disciplines.

## References

- Acar, M., M. T. Ozuledemir, O. Akyilmaz, R. N. Celik and T. Ayan (2006). Deformation analysis with Total Least Squares, *Natural Hazards and Earth System Sciences*, 6, 663-669.
- Achour, H., N. J. V. D. Rebai, Driessche and S. Bouaziz, (2012). Modelling uncertainty of stream networks Derived from elevation data using two free softwares: R and SAGA, *Journal of Geographic Information System*, 4, 153-160.
- Akyilmaz, O., M. T. Ozludemir, T. Ayan and R. N. Celik, (2009). Soft computing methods for geoidal height transformation, *Earth Planets Space*, 61, 825-833.
- Alreja, J., S. Parab, S. Mathur and P. Samui, (2015). Estimating hysteretic energy demand in steel moment resisting frames using multivariate adaptive regression spline and least square support vector machine, *AinsShans Engineering Journal*, doi:<http://dx.doi.org/10.1016/j.as.ej.2014.12.006>, 1-7,
- Annan, R. F., Y. Y. Ziggah, J. Ayer and C. A. Odutola, (2016). Accuracy Assessment of heights obtained from Total station and level instrument using Total Least Squares and Ordinary Least Squares Methods, *Journal of Geomatics and Planning*, 3(2), 87-92.
- Anon. (2013). Nist/Sematech e-Handbook of Statistical Methods, <http://www.itl.nist.gov/div898/handbook/>.
- Ansah, E., (2016). DGPS Networks adjustments using least squares collocation, (Unpublished) BSc Project Work, University of Mines and Technology, Tarkwa, Ghana, pp. 1-57.
- Asklunel, R. and B. Eldvall, (2005). Contamination of water resources in Tarkwamining area of Ghana, (Unpublished) MSc Thesis, Department of Engineering Geology, Royal Institute of Technology, pp. 1-72.
- Bao, H., D. Zhao, Z. Fu, J. Zhu and Z. Gao, (2011). Application of genetic algorithm improved BP neural network in automated deformation monitoring, Seventh International Conference on Natural Computation, Shanghai-China, Institute of Electrical and Electronics Engineers, doi: 10.1109/ICNC.2011.6022149, pp. 1-5.
- Ben-Gal, I. (2005). Outlier Detection, *Data mining and knowledge discovery handbook*, Springer, pp. 131 – 146.
- Carlson, B. and D. Goodman, (2014). Network least Square adjustment demystified, Carlson Incorporated, Accessed: October 24, 2016, 32 pp.
- Craven, P. and G. Wahba, (1979). Smoothing noisy data with spline function: estimating the correct degree of smoothing by the method of generalized cross-validation, *Numerische Mathematik*, 31, pp. 317-403.
- Devi, K. M. and R. Karthikeyan, (2015). A survey on outlier detection for uncertain meteorology data, *International Journal of Engineering Sciences & Research*, 4(12), 203-208.
- Dreiseit, S., and L. Ohno-Machado, (2002). Logistic regression and artificial neural network classification models: A Methodology Review, *Journal of Biomedical Science*, 35(5-6), 352-359.
- Du, S., J. Zhang, Z. Deng and J. Li, (2014). A new approach of geological disasters forecasting using meteorological factors based on genetic algorithm optimized BP neural network, *Elektronika IR Elektro Technika*, 20(4), 57-62.
- Dudek, G. (2011). Generalized regression neural network for forecasting time series with multiple seasonal cycles, *Springer-Verlag Berlin Heidelberg*, 1, pp. 1-8.
- Erdogan, S. (2009). A comparison of interpolation methods for producing digital elevation models at the field scale, *Earth Surface Processes and Landforms*, 34, 366-376.
- Filzmoser, P. (2004). A multivariate outlier detection method, *Proceedings of the Seventh International Conference on Computer Data Analysis and Modeling*, S. Aivazian, P. Filzmoser & Y. Kharin (eds.). Belarusian State University, Minsk, 1, pp. 18 – 22.
- Friedman, J. H. (1991). Multivariate adaptive regression splines, *Annals Statistics*, 19, 1-67.
- Fu, B., and X. Liu, (2014). Application of artificial neural network in GPS height transformation, *Applied Mechanics and Materials*, 501(504), 2162-2165.
- Gauss, C. F. (1823). *Theoria Combinationis observationum erroribus minimis obnoxiae*, Werke, 4, Gottingen, Germany, 1-5.
- Golub, G. H. and C. F. Van Loan, (1980). An analysis of the Total Least Squares problem, *SIAM Journal on Numerical Analysis*, 17(6), 883-893.
- Hannan, S. A., R. R. Manza and R. J. Ramteke, (2010). Generalized regression neural network and radial basis function for heart disease diagnosis, *International Journal of Computer Applications*, 7(13), 7-13.
- Idri, A., A. Zakrani and A. Zahi, (2010). Design of radial basis function neural networks for software effort estimation, *International Journal of Computer Science*, 7(4), 11-17.
- Ismail, S., A. Shabri and R. Samsudin, (2012). A hybrid model of self-organizing maps and least square Support Vector Machine for River flow forecasting, *Hydrological Earth System Science*, 16, 4417-4433.
- Kalooop, M. R., Rabah, M., Hu, J. W. and A. Zaki, (2017). Using advanced soft computing techniques for regional shoreline geoid model estimation and evaluation, *Marine*

- Georesources & Geotechnology, doi: 10.1080/1064119x.2017.1370622, 1-11.
- Kortatsi, B. K. (2004). Hydrochemistry of groundwater in the mining area of Tarkwa-Prestea, Ghana, (Unpublished) PhD Thesis, University of Ghana, Legon-Accra, Ghana, pp. 1-45.
- Kumar, S., A. Shukla and N. Tiwan, (2014). Bayesian analysis of a stationary AR (1) Model and Outlier, *Electronic Journal of Applied Statistical Analysis*, 7, Issue 1, 81-93.
- Kutoglu, H. S. (2006). Artificial neural networks versus surface polynomials for determination of local geoid, 1st International Gravity Symposium, Istanbul, Turkey, pp. 1-6.
- Lall, U., T. Sangoyomi and H. D. I. Abarbanel, (1996). Nonlinear dynamics of the Great Salt Lake: nonparametric short-term forecasting, *Water Resources Research*, 32, 975-985.
- Leathwick, J. R., D. Rowe, J. Richardson, J. Elith and T. Hastie, (2005). Using multivariate adaptive regression splines to predict the distributions of New Zealand's freshwater diadromous fish, *Freshwater Biology*, 50, 2034-2051.
- Liao, D. C., Q. J. Wang, Y. H. Zhou, X. H. Liao and C. L. Huang, (2012). Long-term prediction of the earth orientation parameters by the artificial neural network technique, *Journal of Geodynamics*, 62, 87-92.
- Liu, S., and J., Li and S. Wang, (2011). A hybrid GPS height conversion approach considering of neural network and topographic correction, *International Conference on Computer Science and Network Technology*, China, Institute of Electrical and Electronics Engineers, pp. 1-5, doi: 10.1109/ICCSNT.2011.6182386.
- Maxime, C., V. Thierry and S. Romuald, (2005). Modelling uncertainties in pillar stability analysis, *Post Mining 2005*, November 16-17, Nancy, France, pp. 1-12.
- Peprah, M. S. and I. O. Mensah, (2017). Performance evaluation of the Ordinary Least Square (OLS) and Total Least Square (TLS) in adjusting field data: an empirical study on a DGPS Data, *South African Journal of Geomatics*, 6(1), 73-89.
- Peprah, M. S., Ziggah, Y. Y., and Yakubu, I. (2017). Performance evaluation of the earth gravitational model (EGM2008) – A Case Study, *South African Journal of Geomatics*, 6(1), 47-72.
- Pinho, R., H. Crowley and J. J. Bommer, (2008). Open-source software and treatment of epistemic uncertainties in earthquake loss modelling, *The 14th World Conference on Earthquake Engineering*, October 12-17, 2008, Beijing, China, pp. 1-8.
- Samui, P. (2013). Multivariate Adaptive Regression Spline (MARS) for prediction of elastic modulus of jointed rock mass, *Geotechnical and Geological Engineering*, 31, 249-253.
- Samui, P. and D. P. Kothari, (2012). A multivariate adaptive regression spline approach for prediction of maximum shear modulus (Gmax) and minimum damping ratio ( $\xi_{min}$ ), *Engineering Journal*, 16(5), 1-10.
- Samui, P. and D. Kim, (2012). Modelling of reservoir-induced earthquakes: A multivariate adaptive regression spline, *Journal of Geophysics and Engineering*, 9, 494-497.
- Samui, P. and P. Kurup, (2012). Multivariate Adaptive Regression Spline (MARS) and Least Squares Support Vector Machine (LSSVM) for OCR prediction", *Soft Computing*, 16, 1347-1351.
- Schuessler, N. and K. Axhausen, (2009). Processing raw data from global positioning systems without additional information, *transportation research record*, *Journal of the Transportation Research Board*, 2105, 28 – 36.
- Sharma, P. K., H. Haleem and T. Ahmad, (2015). Improving classification by Outlier Detection and Removal, *Advances in Intelligent Systems and Computing*, 2(338), 621-624.
- Specht, D. (1991). A general regression neural network, *Institute of Electrical and Electronics Engineers Transactions on Neural Networks*, 2(6), 568-576.
- Srichandan, S. (2012). A new approach of software effort estimation using radialbasis function neural networks, *ISSN (Print)*, 1(1), pp. 2319-2526.
- Yakubu, I. and B. Kumi-Boateng, (2011). Control position fix using single frequency global positioning system receiver techniques – A case study, *Environmental and Earth Sciences Research Journal*, 3(1), 32-37.
- Yakubu, I., Y. Y. Ziggah and M. S. Peprah, (2018). Adjustment of DGPS Data using artificial intelligence and classical least square techniques, *Journal of Geomatics*, 12(1), pp. 13-20.
- Yanmin, J., T. Xiaohua and L. Lingyun, (2011). Total least squares with application in geospatial data Processing, *Proceedings of Nineteenth International Conference on Geoinformatics*, Shanghai, China, pp. 1-3.
- Zahra, T. (2010). Quantifying uncertainties in landslide runout modelling, (Unpublished) MSc Thesis, Institute for Geo-Information Science and Earth Observation, Enschede, The Netherlands, pp. 1-94.
- Ziggah, Y. Y., H. Youjian and C. A. Odutola, (2013). Determination of GPS coordinate transformation parameters of geodetic data between reference datums, A case study of Ghana geodetic reference network, *International Journal of Engineering Sciences and Research Technology*, 2(4), 2277-9655.
- Ziggah, Y. Y., H. Youjian, A. Tierra, A. A Konate, and Z. Hui, (2016). Performance evaluation of artificial neural networks for planimetric coordinate transformation-A case study, *Ghana, Arabian Journal of Geosciences*, Vol. 9, pp. 698-714, doi: 10.1007/s12517-016-2729-7.